A Study of the Common Core State Standards

Mathematics

A Study of the Standards: Goal and Expectations

Participants will gain a common understanding of the Common Core State Standards and develop a strong working knowledge of the standards' effect on teaching and learning.

Session participants will learn . . .

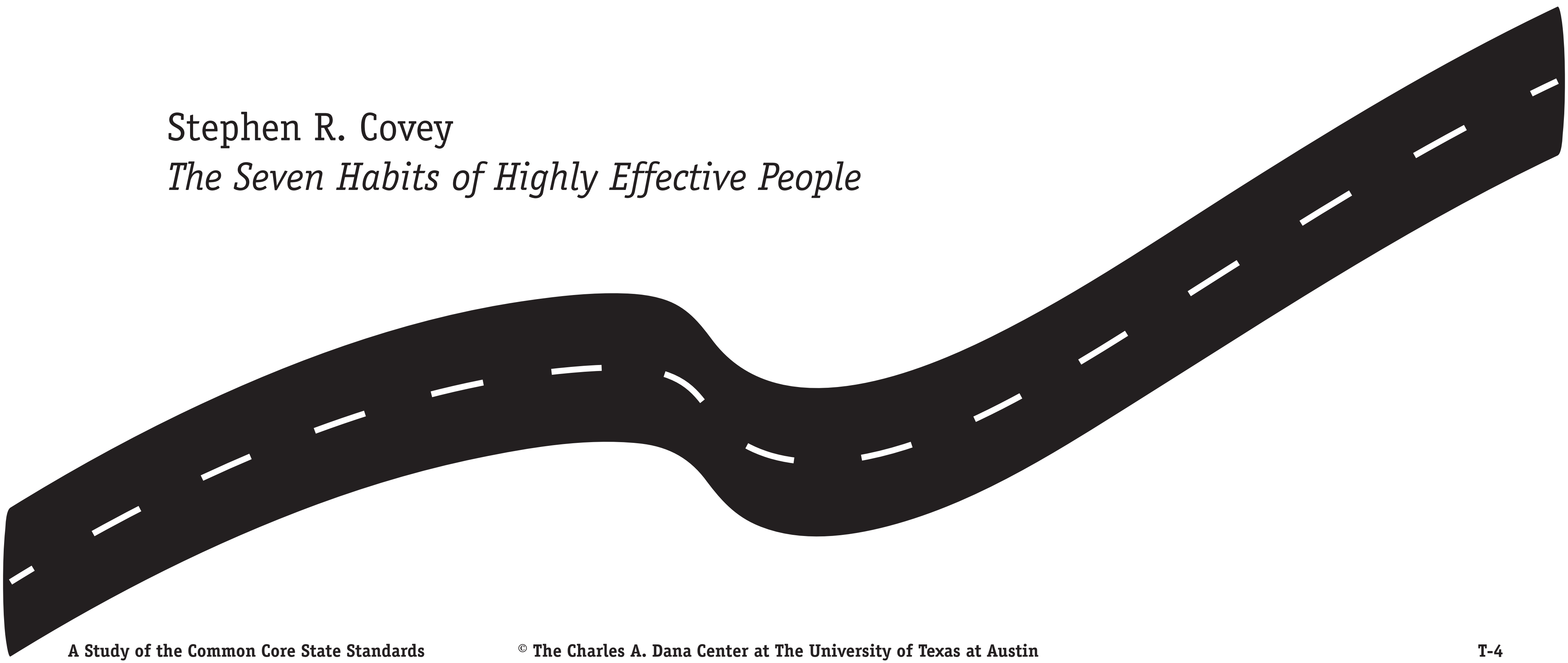
- **how to use a set of structured tools to promote conversations and collaboration around the Common Core State Standards.**
- **how to use the Common Core State Standards to guide decision making about teaching, learning, and assessment.**

Group Norms

- Understand that those who work, learn.
- Phrase questions for the benefit of everyone.
- Recognize that everyone has expertise.
- Challenge ideas, not people.
- Share talk time.

"To begin with the end in mind means to start with a clear understanding of your destination. It means to know where you're going so that you better understand where you are now and so that the steps you take are always in the right direction."

Stephen R. Covey
The Seven Habits of Highly Effective People



Reflect

- **Do you agree or disagree with the central idea of this quotation?**
- **What is the relationship between this quotation and the standards?**

The Common Core State Standards do not provide . . .

- a complete scope and sequence,
- a course outline, or
- *all* the essential skills and knowledge students *could* have.

The Common Core State Standards do . . .

- outline the most important essential skills and knowledge *every* student needs to master to succeed in college and careers.

Common Core State Standards Development

- The Common Core State Standards Initiative is a state-led effort coordinated by the National Governors Association Center for Best Practices (NGA Center) and the Council of Chief State School Officers (CCSSO).
- The standards were developed in collaboration with teachers, school administrators, and experts to provide a clear and consistent framework to prepare our children for college and the workforce.

Common Core State Standards Development *(continued)*

- Aligned with college and work expectations;
- Include rigorous content and application of knowledge through high-order skills;
- Build upon strengths and lessons of current state standards;
- Informed by top-performing countries, so that all students are prepared to succeed in our global economy and society; and
- Evidence and/or research based.

As new research is conducted and implementation of the Common Core State Standards is evaluated, the standards will be revised on a set review cycle.

Structure

The Common Core State Standards for Mathematics are comprised of two corresponding and connected sets of standards:

1. Standards for Mathematical Practice

A set of 8 standards that describe the ways in which the mathematical content standards should be approached.

2. Standards for Mathematical Content

These standards define what students should understand and be able to do in their study of mathematics.

Structure

Standards for Mathematical Practice (K–High School)

Standard title

[ ] Narrative description

Mathematics | Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions,

communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

5 Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8 Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content
The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.

The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.

In this respect, those content standards which set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Standards for Mathematical Practice: K–High School

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

Structure

Standards for Mathematical Content (K–8)

Introduction

- Provides important contextual information and calls out and describes critical areas of focus.

Domain

- Large group of related standards; connects topics and content between and among grade levels.

Clusters/Cluster Heading

- Smaller set of related standards within the domain; identifies the primary idea.

Standards

- Describe what students should know and be able to do for that cluster heading, domain, and grade level.

Structure: K–8 Mathematics Content Standards

Introduction

Domain

Cluster heading

Content standard

Mathematics | Grade 3

In Grade 3, instructional time should focus on four critical areas: (1) developing understanding of multiplication and division and strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with numerator 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes.

(1) Students develop an understanding of the meanings of multiplication and division of whole numbers through activities and problems involving equal-sized groups, arrays, and area models; multiplication is finding an unknown product, and division is finding an unknown factor in these situations. For equal-sized group situations, division can require finding the unknown number of groups or the unknown group size. Students use properties of operations to calculate products of whole numbers, using increasingly sophisticated strategies based on these properties to solve multiplication and division problems involving single-digit factors. By comparing a variety of solution strategies, students learn the relationship between multiplication and division.

(2) Students develop an understanding of fractions, beginning with unit fractions. Students view fractions in general as being built out of unit fractions, and they use fractions along with visual fraction models to represent parts of a whole. Students understand that the size of a fractional part is relative to the size of the whole. For example, 1/2 of the paint in a small bucket could be less paint than 1/3 of the paint in a larger bucket, but 1/3 of a ribbon is longer than 1/5 of the same ribbon because when the ribbon is divided into 3 equal parts, the parts are longer than when the ribbon is divided into 5 equal parts. Students are able to use fractions to represent numbers equal to, less than, and greater than one. They solve problems that involve comparing fractions by using visual fraction models and strategies based on noticing equal numerators or denominators.

(3) Students recognize area as an attribute of two-dimensional regions. They measure the area of a shape by finding the total number of same-size units of area required to cover the shape without gaps or overlaps, a square with sides of unit length being the standard unit for measuring area. Students understand that rectangular arrays can be decomposed into identical rows or into identical columns. By decomposing rectangles into rectangular arrays of squares, students connect area to multiplication, and justify using multiplication to determine the area of a rectangle.

(4) Students describe, analyze, and compare properties of two-dimensional shapes. They compare and classify shapes by their sides and angles, and connect these with definitions of shapes. Students also relate their fraction work to geometry by expressing the area of part of a shape as a unit fraction of the whole.

Grade 3 Overview

Operations and Algebraic Thinking

- **Represent and solve problems involving multiplication and division.**
- **Understand properties of multiplication and the relationship between multiplication and division.**
- **Multiply and divide within 100.**
- **Solve problems involving the four operations, and identify and explain patterns in arithmetic.**

Number and Operations in Base Ten

- **Use place value understanding and properties of operations to perform multi-digit arithmetic.**

Number and Operations—Fractions

- **Develop understanding of fractions as numbers.**

Measurement and Data

- **Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.**
- **Represent and interpret data.**
- **Geometric measurement: understand concepts of area and relate area to multiplication and to addition.**
- **Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**

Geometry

- **Reason with shapes and their attributes.**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Operations and Algebraic Thinking 3.OA

Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5 × 7.*
2. Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.*
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.[1]
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations 8 × ? = 48, 5 = $\square$ ÷ 3, 6 × 6 = ?.*

Understand properties of multiplication and the relationship between multiplication and division.

5. Apply properties of operations as strategies to multiply and divide.[2] *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)*
6. Understand division as an unknown-factor problem. *For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.

7. Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

8. Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.[3]
9. Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

[1] See Glossary, Table 2.
[2] Students need not use formal terms for these properties.
[3] This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order (Order of Operations).

Number and Operations in Base Ten 3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.[4]

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Number and Operations—Fractions[5] 3.NF

Develop understanding of fractions as numbers.

1. Understand a fraction 1/*b* as the quantity formed by 1 part when a whole is partitioned into *b* equal parts; understand a fraction *a*/*b* as the quantity formed by *a* parts of size 1/*b*.
2. Understand a fraction as a number on the number line; represent fractions on a number line diagram.
   a. Represent a fraction 1/*b* on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into *b* equal parts. Recognize that each part has size 1/*b* and that the endpoint of the part based at 0 locates the number 1/*b* on the number line.
   b. Represent a fraction *a*/*b* on a number line diagram by marking off *a* lengths 1/*b* from 0. Recognize that the resulting interval has size *a*/*b* and that its endpoint locates the number *a*/*b* on the number line.
3. Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
   a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
   b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model.
   c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.*
   d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

Measurement and Data 3.MD

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

1. Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.

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[4]A range of algorithms may be used.

[5]Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.

2. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).[6] Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.[7]

Represent and interpret data.

3. Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*
4. Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

5. Recognize area as an attribute of plane figures and understand concepts of area measurement.
   a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
   b. A plane figure which can be covered without gaps or overlaps by *n* unit squares is said to have an area of *n* square units.
6. Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
7. Relate area to the operations of multiplication and addition.
   a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
   b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
   c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
   d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

8. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

[6]Excludes compound units such as cm^3 and finding the geometric volume of a container.
[7]Excludes multiplicative comparison problems (problems involving notions of “times as much”; see Glossary, Table 2).

Geometry 3.G

Reason with shapes and their attributes.

1. Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

2. Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.*

Mathematical Content Standards

K-8 Domains

Kindergarten–Grade 2

- Counting & Cardinality (K only)
- Operations & Alg. Thinking
- Number & Operations in Base 10
- Measurement & Data
- Geometry

Grades 3–5

- Operations & Alg. Thinking
- Number & Operations in Base 10
- Number & Operations–Fractions
- Measurement & Data
- Geometry

Grades 6–7

- Ratios & Proportional Relationships
- Number System
- Expressions & Equations
- Geometry
- Statistics & Probability

Grade 8

- Number System
- Expressions & Equations
- Functions
- Geometry
- Statistics & Probability

Structure

Standards for Mathematical Content (High School)

Conceptual Category

- Provides a coherent view of high school mathematics.

Introduction

- Provides important contextual information.

Domain

- Chunks a large group of related standards; connects topics and content between and among conceptual categories.

Clusters/Cluster Heading

- Smaller sets of related standards within the domain; identifies the primary idea.

Standards

- Describe what students should know and be able to do for that cluster heading, domain, and conceptual category.

Structure: High School Mathematics Content Standards

Conceptual category

Introduction

Domain

Cluster heading

Content standard

Mathematics Standards for High School

The high school standards specify the mathematics that all students should study in order to be college and career ready. Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+), as in this example:

> (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers).

All standards without a (+) symbol should be in the common mathematics curriculum for all college and career ready students. Standards with a (+) symbol may also appear in courses intended for all students.

The high school standards are listed in conceptual categories:

- Number and Quantity
- Algebra
- Functions
- Modeling
- Geometry
- Statistics and Probability

Conceptual categories portray a coherent view of high school mathematics; a student's work with functions, for example, crosses a number of traditional course boundaries, potentially up through and including calculus.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group.

Mathematics | High School—Functions

Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models.

In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = 100/v$ expresses this relationship algebraically and defines a function whose name is T.

The set of inputs to a function is called its domain. We often infer the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context.

A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Functions presented as expressions can model many important phenomena. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. Linear functions with a constant term of zero describe proportional relationships.

A graphing utility or a computer algebra system can be used to experiment with properties of these functions and their graphs and to build computational models of functions, including recursively defined functions.

Connections to Expressions, Equations, Modeling, and Coordinates.

Determining an output value for a particular input involves evaluating an expression; finding inputs that yield a given output involves solving an equation. Questions about when two functions have the same value for the same input lead to equations, whose solutions can be visualized from the intersection of their graphs. Because functions describe relationships between quantities, they are frequently used in modeling. Sometimes functions are defined by a recursive process, which can be displayed effectively using a spreadsheet or other technology.

Functions Overview

Interpreting Functions

- **Understand the concept of a function and use function notation**
- **Interpret functions that arise in applications in terms of the context**
- **Analyze functions using different representations**

Building Functions

- **Build a function that models a relationship between two quantities**
- **Build new functions from existing functions**

Linear, Quadratic, and Exponential Models

- **Construct and compare linear, quadratic, and exponential models and solve problems**
- **Interpret expressions for functions in terms of the situation they model**

Trigonometric Functions

- **Extend the domain of trigonometric functions using the unit circle**
- **Model periodic phenomena with trigonometric functions**
- **Prove and apply trigonometric identities**

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Interpreting Functions — F-IF

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.

2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by* $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$.

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★

6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
   a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
   b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
   c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
   d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
   e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
   a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
   b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.*

9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities

1. Write a function that describes a relationship between two quantities.*
   a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
   b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
   c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.*

Build new functions from existing functions

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*
4. Find inverse functions.
   a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example,* $f(x) = 2x^3$ *or* $f(x) = (x+1)/(x–1)$ *for* $x \neq 1$.
   b. (+) Verify by composition that one function is the inverse of another.
   c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
   d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
5. (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Linear, Quadratic, and Exponential Models* F-LE

Construct and compare linear, quadratic, and exponential models and solve problems

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
   a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
   b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
   c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model

5. Interpret the parameters in a linear or exponential function in terms of a context.

Trigonometric Functions — F-TF

Extend the domain of trigonometric functions using the unit circle

1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Model periodic phenomena with trigonometric functions

5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★
6. (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
7. (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

Prove and apply trigonometric identities

8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Mathematics | High School—Modeling

Modeling links classroom mathematics and statistics to everyday life, work, and decision-making. Modeling is the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions. Quantities and their relationships in physical, economic, public policy, social, and everyday situations can be modeled using mathematical and statistical methods. When making mathematical models, technology is valuable for varying assumptions, exploring consequences, and comparing predictions with data.

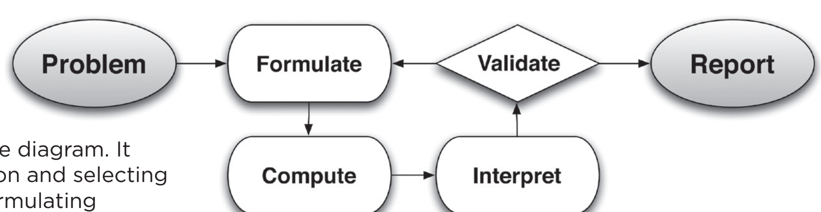
A model can be very simple, such as writing total cost as a product of unit price and number bought, or using a geometric shape to describe a physical object like a coin. Even such simple models involve making choices. It is up to us whether to model a coin as a three-dimensional cylinder, or whether a two-dimensional disk works well enough for our purposes. Other situations—modeling a delivery route, a production schedule, or a comparison of loan amortizations—need more elaborate models that use other tools from the mathematical sciences. Real-world situations are not organized and labeled for analysis; formulating tractable models, representing such models, and analyzing them is appropriately a creative process. Like every such process, this depends on acquired expertise as well as creativity.

Some examples of such situations might include:

- Estimating how much water and food is needed for emergency relief in a devastated city of 3 million people, and how it might be distributed.
- Planning a table tennis tournament for 7 players at a club with 4 tables, where each player plays against each other player.
- Designing the layout of the stalls in a school fair so as to raise as much money as possible.
- Analyzing stopping distance for a car.
- Modeling savings account balance, bacterial colony growth, or investment growth.
- Engaging in critical path analysis, e.g., applied to turnaround of an aircraft at an airport.
- Analyzing risk in situations such as extreme sports, pandemics, and terrorism.
- Relating population statistics to individual predictions.

In situations like these, the models devised depend on a number of factors: How precise an answer do we want or need? What aspects of the situation do we most need to understand, control, or optimize? What resources of time and tools do we have? The range of models that we can create and analyze is also constrained by the limitations of our mathematical, statistical, and technical skills, and our ability to recognize significant variables and relationships among them. Diagrams of various kinds, spreadsheets and other technology, and algebra are powerful tools for understanding and solving problems drawn from different types of real-world situations.

One of the insights provided by mathematical modeling is that essentially the same mathematical or statistical structure can sometimes model seemingly different situations. Models can also shed light on the mathematical structures themselves, for example, as when a model of bacterial growth makes more vivid the explosive growth of the exponential function.



The basic modeling cycle is summarized in the diagram. It involves (1) identifying variables in the situation and selecting those that represent essential features, (2) formulating a model by creating and selecting geometric, graphical, tabular, algebraic, or statistical representations that describe relationships between the variables, (3) analyzing and performing operations on these relationships to draw conclusions, (4) interpreting the results of the mathematics in terms of the original situation, (5) validating the conclusions by comparing them with the situation, and then either improving the model or, if it is acceptable, (6) reporting on the conclusions and the reasoning behind them. Choices, assumptions, and approximations are present throughout this cycle.

In descriptive modeling, a model simply describes the phenomena or summarizes them in a compact form. Graphs of observations are a familiar descriptive model—for example, graphs of global temperature and atmospheric CO_2 over time.

Analytic modeling seeks to explain data on the basis of deeper theoretical ideas, albeit with parameters that are empirically based; for example, exponential growth of bacterial colonies (until cut-off mechanisms such as pollution or starvation intervene) follows from a constant reproduction rate. Functions are an important tool for analyzing such problems.

Graphing utilities, spreadsheets, computer algebra systems, and dynamic geometry software are powerful tools that can be used to model purely mathematical phenomena (e.g., the behavior of polynomials) as well as physical phenomena.

Modeling Standards *Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol* (★).

Mathematical Content Standards

High school conceptual categories and domains

Number and Quantity

The Real Number System

Quantities

The Complex Number System

Vector and Matrix Quantities

Algebra

Seeing Structure in Expressions

Arithmetic with Polynomials and Rational Expressions

Creating Equations

Reasoning with Equations and Inequalities

Functions

Interpreting Functions

Building Functions

Linear, Quadratic, and Exponential Models

Trigonometric Functions

Modeling

Mathematical Content Standards *(continued)*

High school conceptual categories and domains

Geometry

Congruence

Similarity, Right Triangles, and Trigonometry

Circles

Expressing Geometric Properties with Equations

Geometric Measurement and Dimension

Modeling with Geometry

Statistics and Probability

Interpreting Categorical and Quantitative Data

Making Inferences and Justifying Conclusions

Conditional Probability and the Rules of Probability

Using Probability to Make Decisions

Structure

What? *What did you learn as a result of the structure activity?*

So what? *What is important about what you have learned?*

Now what? *What actions will you take as a result of your learning?*

Mathematics Appendix

appendix a:

Designing high school mathematics courses based on the common core state standards

Outlines four model course pathways

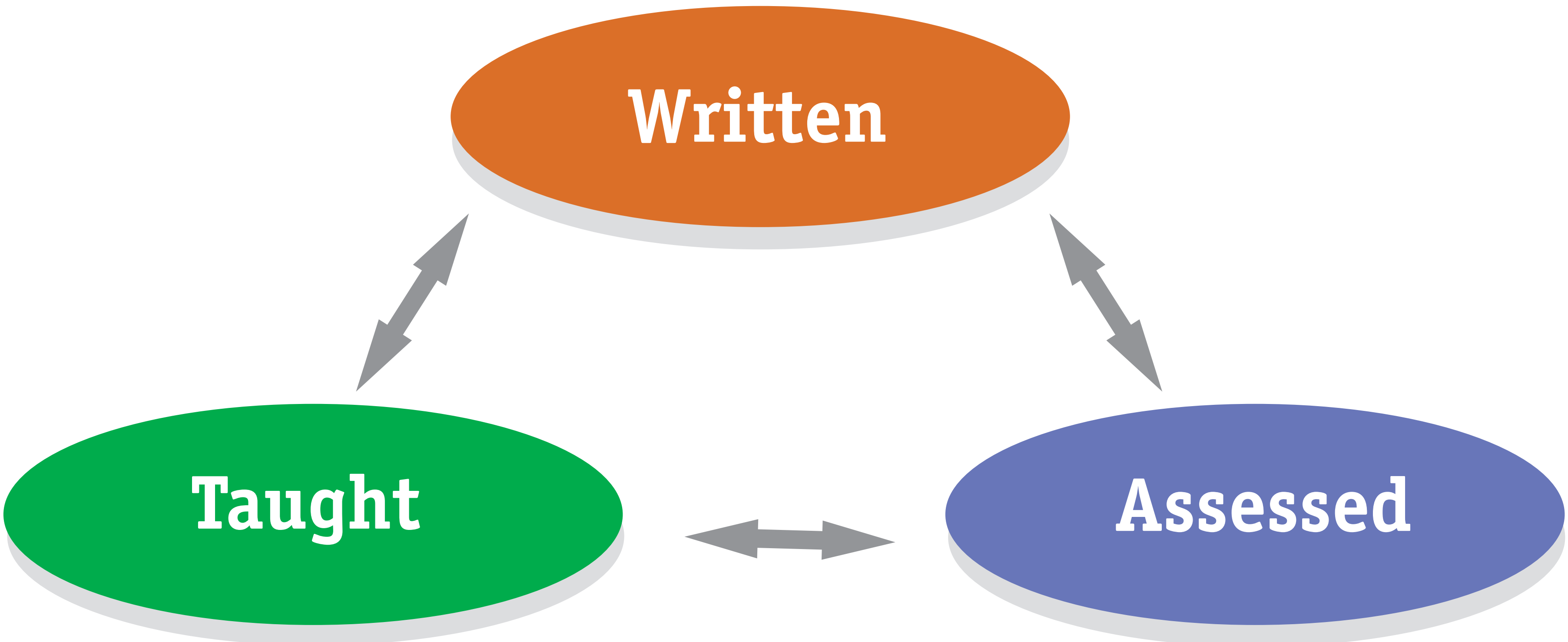
- Traditional
- Integrated
- Compacted of Traditional
- Compacted of Integrated

Alignment Is More Than . . .

- A chart
- A textbook correlation
- A scope and sequence
- A curriculum guide
- A testing plan

These things imply alignment, but they do not give us alignment.

A Basic Alignment Principle



Adapted from the work of Fenwick English

Alignment Means *Every* Educator . . .

- Understands what is expected of students.
- Understands these expectations within the context of the K-12 program.
- Accepts responsibility for these expectations.

Understanding Alignment

An investigation activity

- It is not about developing content knowledge. It is about learning a process to understand alignment and its implications for teaching and learning.
- It is not about demonstrating our content knowledge. It is about engaging in a collaborative process and constructing meaning using that process.
- It is not about specific grade-level content. It is about developing a K–12 perspective on alignment.
- It is not about "the product". It is about collegial conversations focused on the standards.

Understanding Alignment

Investigating learning trajectories

Big Idea: ____

K.MD

Classify objects and count # of objects in each category

- Classify obj. in given categories
- Count # of objects in each category and sort category by count
- limit count to less than or equal to 10

1.MD Represent and interpret data

- Organize, represent, interpret data up to 3 cat.
- Ask and answer questions about total # data points (how many each category more/less)
- Not limited to 10 for count

K.MD Classify objects and count # of objects in each category

- Classify obj. in given categories
- Count # of objects in each category and sort category by count
- limit count to less than or equal to 10

2.MD Represent and interpret data

- Generate measurement data by measuring lengths
- Show measurements by making line plot marked off in whole #s
- Draw picture & bar graph (single unit scale) to represent data up to 4 categories
- Solve +/- & compare problems using bar graph

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- Organize, represent, interpret data up to 3 cat.
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K.MD Classify objects and count # of objects in each category

- Classify obj. in given categories
- Count # of objects in each category and sort category by count
- limit count to less than or equal to 10

3.MD Represent + interpret data

- draw scaled picture and bar graph to represent data set with several categories
- solve 1+2 step "how many more" "how many less" from scaled bar graphs.
- generate measurement data by measuring lengths using halves + fourths
- Show data by making line plot where horizontal scale is marked off in appropriate units.

2.MD Represent and interpret data

- Generate measurement data by measuring lengths
- Show measurements by making line plot marked off in whole #s
- Draw picture + bar graph (single unit scale) to represent data up to 4 categories
- Solve +/- + Compare problems using bar graph

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- Not limited to 10 for count

K.MD Classify objects and count # of objects in each category

- Classify obj. in given categories
- Count # of objects in each category and sort category by count
- limit count to less than or equal to 10

4.MD Represent + Interpret data

- make line plot to display data set of measurements halves, fourths, eighths
- solve problems involving +/- of fractions by using info from line plots
- No mention of bar, picture, scale

3.MD Represent + interpret data

- draw scaled picture and bar graph to represent data set with several categories
- solve 1+2 step "how many more" "how many less" from scaled bar graphs.
- generate measurement data by measuring lengths using halves + fourths
- show data by making line plot where horizontal scale is marked off in appropriate units.

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Understanding Alignment

Directions for investigating learning trajectories

As a table group . . .

1. **Determine what your big idea means.**

2. **Read, discuss, and come to consensus on what the standards say students need to know and be able to do. Consider all parts of the standards.**

3. **Analyze how the demands of the standard change between grade levels. Consider changes in content and processes.**

4. **Document your findings.**

Understanding Alignment

Investigating learning trajectories

Represent and interpret data	2D and 3D geometry	Addition and subtraction
K.MD.3	K.G.2; K.G.3	K.OA.1; K.OA.2; K.OA.5
1.MD.4	1.G.1; 1.G.2	1.OA.6; 1.NBT.4; 1.NBT.5; 1.NBT.6
2.MD.9; 2.MD.10	2.G.1	2.OA.2; 2.NBT.5; 2.NBT.6; 2.NBT.7
3.MD.3; 3.MD.4	3.G.1	3.NBT.2
4.MD.4	4.G.1	4.NBT.4; 4.NF.3c
5.MD.2	5.G.3; 5.G.4	5.NBT.7; 5.NF.1
6.SP.4	6.G.4	6.NF.3
7.SP.8b	7.G.3	7.NS.1d
8.SP.1; 8.SP.3; 8.SP.4	8.G.4	Grade 8—none
S-ID.1 through 9	G-MD.4	N-CN.2; N-VM.4a–c; N-VM.8; A-APR.1; A-APR.7

Understanding Alignment

Investigating learning trajectories *(continued)*

Area and perimeter	Place value
2.G.2	K.NBT.1
3.MD.5a-b; 3.MD.6; 3.MD.7a,b,d;	1.NBT.2a; 1.NBT.2b; 1.NBT.2c; 1.NBT.3
3MD.8	2.NBT.1a; 2.NBT.1b
4.MD.3	3.NBT.1
Grade 5—none	4.NBT.2; 4.NBT.3
6.G.1	5.NBT.1; 5.NBT.4
7.G.1; 7.G.4	Grade 6—none
Grade 8—none	Grade 7—none
G-GEP.7; G-MF.2	Grade 8—none

Understanding Alignment: Reflection

1. How can the learning from this investigation affect the classroom teacher?

2. How can the learning from this investigation affect the conversations at the grade or department level?

3. How can the learning from this investigation affect the conversations at the school and district level?

4. How can the learning from this investigation guide our work toward our goals?

5. How might you use this investigation activity back on your campus and in your schools?

Three Levels of Instruction with Supporting Activities

Step 1: Provide Developmental Activities

- Emphasize problem solving.
- Use interesting problems to frame and motivate exploration.
- Use problem situations that relate to the lives of your students.
- Guide student thinking using questions.
- Do not answer your own questions. Give students time to answer.
- Use models that can be manipulated and studied.
- Emphasize concrete objects and pictures before introducing symbols.
- Work along with students, observing their progress carefully.
- Concentrate on preventing misconceptions instead of correcting them.
- Give corrective feedback as quickly as possible.
- Use observation and oral questions to evaluate, rather than just pencil and paper tasks.

Step 2: Provide Reinforcement Activities

- Create stimulating explorations that build upon previous developmental lessons where students worked together.
- Expand upon the activities that you started in the developmental lessons.
- Use materials in a variety of ways to connect concrete models, pictures, and symbolic representations.
- Emphasize problem solving.
- Organize small cooperative groups where students can share ideas and help each other.
- Let students in small groups take responsibility for making presentations, explaining processes, and creating problems.
- Let students work together but also provide opportunities to work alone.
- Prepare problem solving bulletin boards and learning centers.

Step 3: Provide Drill and Practice Activities

- Create stimulating games where students work together.
- Change the directions for worksheets to create interesting puzzles and explorations.
- Emphasize problem solving.
- Use problem situations to motivate practice.
- Give short sets of exercises and evaluate student progress.
- Do not give long and tedious assignments in which students might practice their own misconceptions.
- Never introduce drill before proper concept development and reinforcement of concepts have taken place.

Gagné, R.M. and Briggs L.J. (1997) *Principles of instructional design* (2nd edition). New York: Holt, Rinehart, and Winston.

Instructional Alignment Chart

*Strand(s):*______________________________

Standard(s) for Grade/Course:	*Standard(s) for Grade/Course:*	*Standard(s) for Grade/Course:*

Changes	*Changes*

Levels of instruction

Implications for instruction and assessment

Instructional Alignment Chart

Strand(s): Represent and interpret data

Standard(s) for Grade/Course: Grade 3	*Standard(s) for Grade/Course:* Grade 4	*Standard(s) for Grade/Course:* Grade 5
3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. 3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves, or quarters	4.MD.4 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots.	5.MD.2 Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots.

Changes	*Changes*

Levels of instruction

Implications for instruction and assessment

Instructional Alignment Chart

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Changes	*Changes*
• Added eighths when making, gathering, and displaying line plot data • Scaled picture and bar graphs disappear • Moved from generating measurement data to solving problems using information from the line plot. • Added solving problems using line plots involving adding and subtracting fractions	• Added multiplication and division of fractions when solving problems using line plots

Levels of instruction

Implications for instruction and assessment

Instructional Alignment Chart

Strand(s): Represent and interpret data

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Levels of instruction

- Concept of ½, ¼, ⅛ should be taught at the reinforcement level since it first appeared in grade 3
- Line plots should be instructed at the drill/practice level
- Solving of problems involving adding/subtracting fractions should begin at developmental and moves to reinforcement

Implications for instruction and assessment

Instructional Alignment Chart

Strand(s): Represent and interpret data

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Implications for instruction and assessment

- Use what students know about number lines when creating the line plot for the measurement data
- Use number lines to help students subdivide for halves and fourths to get to eighths.
- Consider the learnings students are engaged in the Domain of Number and Operations—Fractions when planning.
- Identify the Standards for Mathematical Practice that will be used to approach the content.

Instructional Alignment Chart

*Strand(s):*____________________________

Standard(s) for Grade/Course:	*Standard(s) for Grade/Course:*	*Standard(s) for Grade/Course:*

Changes	*Changes*

Levels of instruction

Implications for instruction and assessment

Instructional Alignment Chart

Step 1

Standards for the grade/course______________________________

Purpose:

To develop a common understanding of what the written grade level/course standard(s) require students to know and be able to do.

Process:

Individually, read and analyze the grade/course level standard to determine what is required of students in terms of content and processes.

As a group, discuss and come to consensus on what the grade/course level standard(s) require students to know and be able to do.

Instructional Alignment Chart

Step 2

Changes

Purpose:

To develop a common understanding of how the standards from adjacent grades affect/influence our understanding of the content and processes of the grade/course level standards.

Process:

Individually, analyze the standards for the adjacent grade levels to determine the similarities and differences. Describe and document the changes between the target grade level and the grade before; the target grade level and the grade after in terms of content and processes.

As a group, discuss and come to consensus on what the grade/course level standard(s) require students to know and be able to do.

Instructional Alignment Chart

Step 3

Levels of Instruction

Purpose:
To identify the intended level of instruction.

Process:
Individually, analyze the changes documented above to determine the appropriate level(s) of instruction (developmental, reinforcement, and/or drill and practice).

As a group, discuss and come to consensus about the appropriate levels of instruction. Document your findings.

Instructional Alignment Chart

Step 4

Implications for Instruction and Assessment

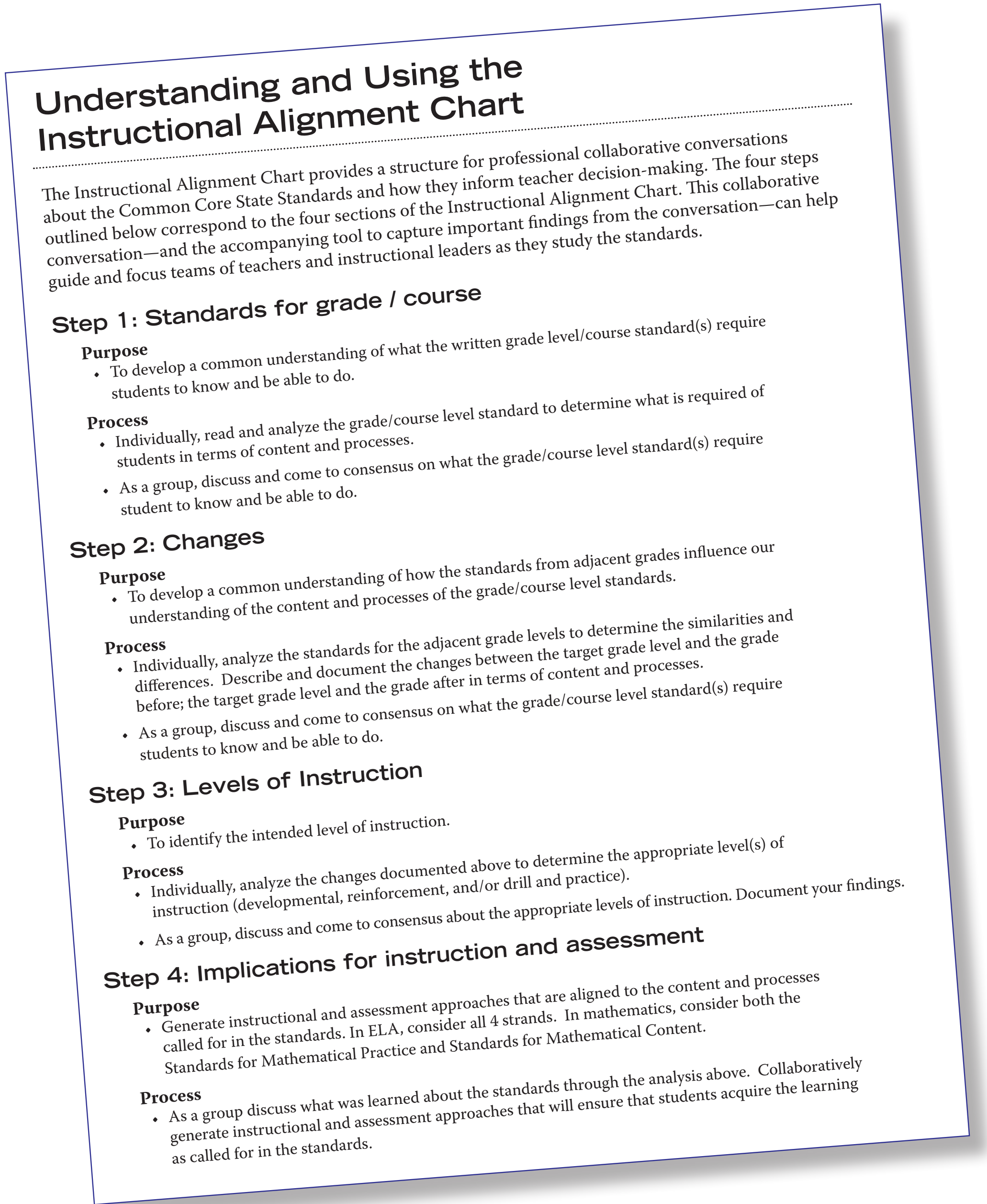
Purpose:

Generate instructional and assessment approaches that are aligned to the content and processes called for in the standards. In ELA, consider all 4 strands. In mathematics, consider both the Standards for Mathematical Practice and Standards for Mathematical Content.

Process:

As a group, discuss what was learned about the standards through the analysis above. Collaboratively generate instructional and assessment approaches that will ensure that students acquire the learning called for in the standards.

Understanding & Using the Instructional Alignment Chart



Understanding and Using the Instructional Alignment Chart

The Instructional Alignment Chart provides a structure for professional collaborative conversations about the Common Core State Standards and how they inform teacher decision-making. The four steps outlined below correspond to the four sections of the Instructional Alignment Chart. This collaborative conversation—and the accompanying tool to capture important findings from the conversation—can help guide and focus teams of teachers and instructional leaders as they study the standards.

Step 1: Standards for grade / course

Purpose
- To develop a common understanding of what the written grade level/course standard(s) require students to know and be able to do.

Process
- Individually, read and analyze the grade/course level standard to determine what is required of students in terms of content and processes.
- As a group, discuss and come to consensus on what the grade/course level standard(s) require student to know and be able to do.

Step 2: Changes

Purpose
- To develop a common understanding of how the standards from adjacent grades influence our understanding of the content and processes of the grade/course level standards.

Process
- Individually, analyze the standards for the adjacent grade levels to determine the similarities and differences. Describe and document the changes between the target grade level and the grade before; the target grade level and the grade after in terms of content and processes.
- As a group, discuss and come to consensus on what the grade/course level standard(s) require students to know and be able to do.

Step 3: Levels of Instruction

Purpose
- To identify the intended level of instruction.

Process
- Individually, analyze the changes documented above to determine the appropriate level(s) of instruction (developmental, reinforcement, and/or drill and practice).
- As a group, discuss and come to consensus about the appropriate levels of instruction. Document your findings.

Step 4: Implications for instruction and assessment

Purpose
- Generate instructional and assessment approaches that are aligned to the content and processes called for in the standards. In ELA, consider all 4 strands. In mathematics, consider both the Standards for Mathematical Practice and Standards for Mathematical Content.

Process
- As a group discuss what was learned about the standards through the analysis above. Collaboratively generate instructional and assessment approaches that will ensure that students acquire the learning as called for in the standards.

Reflection

1. What is the purpose of the Instructional Alignment Chart?
2. Why spend time with your colleagues using the Instructional Alignment Chart?
3. What's the difference between the "building the learning trajectories" and the Instructional Alignment Chart? When would you use each?

A Study of the Standards: Goal and Expectations

Participants will gain a common understanding of the Common Core State Standards and develop a strong working knowledge of the standards' effects on teaching and learning.

Session participants will learn . . .

- **how to use a set of structured tools to promote conversations and collaboration around the Common Core State Standards.**
- **how to use the Common Core State Standards to guide decision making about teaching, learning, and assessment.**