Advanced Mathematical Decision Making Course Materials
Unit Overviews for Pilot Use
July 17, 2009

Advanced Mathematical Decision Making is proposed as a fourth-year course to follow Algebra II. Its primary purpose is to prepare students for college majors that are not math intensive, for technical training, or for a range of career options. This course may also be useful to other students as an elective. Basic course instructional materials and supporting professional development are being prepared for piloting during 2009–2010, with full implementation in 2010–2011. For more information, go to www.utdanacenter.org/amdm.

Introduction

This document constitutes the draft unit overviews for Advanced Mathematical Decision Making (AMDM). This course is made up of nine core units and one supplemental unit; within each unit, there are three to five sections. Each unit requires 2 to 3 weeks of instructional time. AMDM is structured as follows:

- **Unit I:** Analyzing Numerical Data (3 weeks)
  - Using Ratios, Rates, and Percents
  - Using Averages, Weighted Averages, and Indices
  - Estimating Large Numbers
  - Using Check Digits to Validate Identification Numbers
- **Unit II:** Probability (3 weeks)
  - Determining Probabilities
  - Everyday Decisions Based on Probabilities
  - Expected Value
- **Unit III:** Analyzing Statistical Studies (3 weeks)
  - Case Study Analysis
  - Univariate Data
  - Bivariate Data
- **Unit IV:** Designing a Study (3 weeks)
  - Statistical Investigations
  - Statistical Analyses
  - Sources of Variability
- **Unit V:** A Discrete Look at Change (3 weeks)
  - Linear Models
  - Exponential and Logistic Models
  - Cyclical Models
- **Unit VI:** More Models of Change (3 weeks)
  - Review of Recursive Function Models and Their Associated Continuous Models
  - Models of Cyclical Phenomena
  - Step and Piecewise Models
  - Making Decisions and the Strength of a Model

- **Unit VII:** Spatial and Geometric Modeling (3 weeks)
  - Vectors
  - Inaccessible Distances
  - Transformations
- **Unit VIII:** Networks and Graphs (3 weeks)
  - Circuits and Paths
  - Spanning Trees
  - Graph Colorings
  - Graph Structure
  - Program Evaluation and Review Technique Charts
- **Unit IX:** Decision Making in Finance (2 weeks)
  - Future Value of an Investment
  - Present Value of an Investment
  - Annuities
  - Credit Card Debt
  - Comparing Automobile Loan Proposals (optional)
- **Unit X:** Decision Making in Voting and Selection Processes (supplemental, 2 weeks, to be developed in Fall 2009)
  - Division Techniques
  - Using Ranking Methods
  - Using Voting Methods

AMDM Units

Unit I: Analyzing Numerical Data (3 weeks)

Mathematics Overview

This unit focuses on deepening the student's understanding of proportional reasoning and basic numerical calculations such as ratios, rates, and percents by applying them to settings in business, media, consumer, and other areas. By working with familiar mathematical tools, students improve their ability to solve problems by applying appropriate strategies.

In Grades 6 through 8, students learned about using ratios to describe direct proportional relationships involving number, geometry, measurement, and probability. In Grade 6, the emphasis was on using ratios to describe and make predictions in proportional situations and to represent ratios and percents with concrete models, fractions, and decimals. In Grade 7, students estimated and found solutions to application problems involving percent and proportional relationships such as similarity, scaling, unit costs, and related measurement units. In Grade 8, students worked with both proportional and nonproportional linear relationships and continued to estimate and solve application problems involving percents. As students progressed through Algebra I, Algebra II, and Geometry, they continued to gain experience with proportional linear relationships.

The *Analyzing Numerical Data* unit builds upon students' prior knowledge of ratio and focuses on how to make decisions in everyday situations after analyzing information. Using contextual situations, students develop skills that they can apply outside the classroom.

Learning Expectations Addressed

(DM.1) **Analyzing numerical data**. The student analyzes numerical data in everyday situations using a variety of quantitative measures.

The student is expected to:

(A) apply, compare, and contrast ratios, rates, and ratings (such as aspect ratios, growth rates, television program ratings, NFL quarterback ratings, and job ratings) to make informed decisions;

(B) apply, compare, and contrast averages, weighted averages, and indices (such as grade point average, body mass index, NFL quarterback ratings, and Consumer Price Index) to make informed decisions;

(C) solve problems involving large quantities (such as estimating crowd size, counting the number of available phone numbers, estimating animal populations, or managing natural resources); and

(D) apply algorithms to determine the check digit for identification numbers (such as universal product codes [UPCs], vehicle identification numbers [VINs], and credit card numbers) and identify errors in recording and transmitting these numbers.

Section Overviews

The *Analyzing Numerical Data* unit is divided into four sections as outlined below. The activities in each section take two to four instructional days.

Section A: Using Ratios, Rates, and Percents
Students apply proportional reasoning with ratios, rates, and percents to real-world problems involving TV ratings and shares; aspect ratios in airplane wings, photographs, tires, and movies shown on television; growth rates; NFL quarterback ratings; Q ratings for entertainers; and so forth.

Section B: Using Averages, Weighted Averages, and Indices
Students use averages and indices as a tool for finding a weighted average of the criteria used to determine which car or camera to buy and for analyzing the Fan Cost Index for attending sporting events, the Gunning Fog Index for measuring reading difficulty, the Consumer Price Index, and so forth.

Section C: Estimating Large Numbers
Students use various numerical techniques to estimate large numbers in situations such as calculating the number of possible telephone numbers in the United States to see when the numbers will run out, assessing the size of the crowd at a political rally, appraising the number of hamburgers sold by a national fast-food chain in a typical year, and so forth.

Section D: Using Check Digits to Validate Identification Numbers
Students learn how identification numbers such as UPCs, credit card numbers, and VINs are created and how check digits are used to detect errors and prevent fraud.

Unit II: Probability (3 weeks)

Mathematics Overview

This unit focuses on the analysis of information using probability to make decisions about everyday situations. After determining the probability of various events, students expand their knowledge toward making decisions about risks and mathematical fairness of these events.

In Grades 6 through 8, students learned about concepts of probability, how to apply these concepts in both theoretical and experimental situations, and how to use these concepts to make predictions. In grade 6, the emphasis was on constructing sample spaces and tree diagrams and finding probabilities of simple events and their complements. In Grade 7, students extended their construction of sample spaces from simple to composite experiments and found the probability of independent events. In Grade 8, the probability extensions included finding probability of dependent events to simulating events using models. As students progressed through Algebra I, Algebra II, and Geometry, there were no additional probability learning expectations.

The *Probability* unit builds upon students' prior knowledge of probability and focuses on how to make decisions in everyday situations after analyzing information. By using contextual situations, students develop skills that they can apply outside the classroom.

Learning Expectations Addressed

(DM.2) **Analyzing information using probability.** The student analyzes and evaluates risk and return in the context of everyday situations.

The student is expected to:

(A) determine conditional probabilities and probabilities of compound events by constructing and analyzing representations (including tree diagrams, Venn diagrams, and area models) to make decisions in problem situations;

(B) use probabilities to make and justify decisions about risks in everyday life (such as investing in the stock market, taking medication, or selecting car insurance); and

(C) calculate expected value to analyze mathematical fairness, payoff, and risk.

Section Overviews

The *Probability* unit is divided into three sections as outlined below. The activities in each section take three to five instructional days.

Section A: Determining Probabilities
Students construct and analyze representations of events, such as tree diagrams, to determine conditional probabilities. They construct Venn diagrams and determine probabilities of compound events in order to make a decision about the risks involved in the situation.

Section B: Everyday Decisions Based on Probabilities
Students explore the use of probabilities in everyday situations such as taking medication or selecting car insurance. From these explorations, they make a decision and justify that decision about the risk involved in the situation.

Section C: Expected Value
Students calculate expected values to analyze payoffs in a variety of situations, including games, lottery events, and insurance policies. They apply their understanding of expected values to determine the mathematical fairness of situations.

Unit III: Analyzing Statistical Studies (3 weeks)

Mathematics Overview

This unit focuses on the development of background statistical knowledge through the use of case studies. Students build the skills and vocabulary necessary to analyze and critique reported statistical information, summaries, and graphical displays; they prepare oral and written reports of these analyses.

In Grades 6 through 8, students learned about using statistical representations to analyze data. In Grade 6, the emphasis was on using different graphical representations to display the same data (including line plots, line graphs, bar graphs, stem-and-leaf plots, and circle graphs); using mean, median, mode, and range as measures of center and spread; and collecting, organizing, displaying, and interpreting data. In Grade 7, students were expected to choose the appropriate display and justify their choice, and Venn diagrams were introduced. Students also provided convincing arguments based on an analysis of data. In Grade 8, students chose the appropriate measure of center and spread for a data set and justified their choice. They drew conclusions and made predictions by analyzing trends in scatterplots. Students added box-and-whisker plots and histograms to their repertoire of graphical representations. Students in Grade 8 were introduced to the concepts of sampling methods and their effects on validity, and they investigated misuses of statistical information. There were no additional statistical learning expectations in Algebra I, Algebra II, and Geometry.

The *Analyzing Statistical Studies* unit builds upon students' prior knowledge of statistics to ensure that they become more discerning consumers of statistics in everyday situations. This unit provides the foundation for the following unit, which has students design and implement their own statistical studies.

Learning Expectations Addressed

(DM.3) **Critiquing applications of statistics.** The student makes decisions based on understanding, analysis, and critique of reported statistical information and statistical summaries.

The student is expected to:

(A) identify limitations or lack of information in studies reporting statistical information, especially when studies are reported in condensed form;

(B) interpret and compare the results of poll(s) given a margin of error;

(C) identify uses and misuses of statistical analyses in studies reporting statistics or using statistics to justify particular conclusions, including assertions of cause and effect rather than correlation; and

(D) describe strengths and weaknesses of sampling techniques, data and graphical displays, and interpretations of summary statistics and other results appearing in a study, including reports published in the media.

(DM.5) **Communicating statistical information.** The student communicates the results of reported and student-generated statistical studies.

The student is expected to:

(A) report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays, and interpreting results in terms of the question studied;

(B) justify the design and the conclusion(s) of statistical studies, including the methods used for each; and

(C) communicate statistical results in both oral and written formats using appropriate statistical and nontechnical language.

Section Overviews

The *Analyzing Statistical Studies* unit is divided into three sections as outlined below. The activities in each section take three to five instructional days.

Section A: Case Study Analysis
Students are introduced to case studies in order to build background knowledge. They determine whether or not the studies are observational or experimental and identify the sampling techniques used. Students analyze results, including margin of error in opinion polls; they then identify strengths and weaknesses in the study and the presentation of results.

Section B: Univariate Data
Students focus on univariate data, identifying the variable of interest. They interpret a variety of graphical displays and estimate center, spread, shape, and unusual features. Students analyze the appropriateness and usefulness of the chosen measure of center and graphical display. They

identify limitations, lack of information, and possible misinterpretations in media reports. Students then prepare more appropriate reports
for a given audience.

Section C: Bivariate Data
Students focus on bivariate data, identifying the variables of interest. They analyze graphical displays, describing form, direction, and relative linear strength. Students also identify uses and misuses of statistical analyses, including assertions of cause and effect rather than correlation. They then prepare more appropriate reports for a given audience.

Unit IV: Designing a Study (3 weeks)

Mathematics Overview

This unit focuses on introducing students to the basic components of the design and implementation of statistical studies. After collecting and displaying data, students explore introductory techniques of statistical analysis.

In Grades 6 through 8, students learned about statistical representations to analyze data. In Grade 6, the emphasis was on using different graphical representations to display the same data (including line plots, line graphs, bar graphs, stem-and-leaf plots, and circle graphs); using mean, median, mode, and range as measures of center and spread; and collecting, organizing, displaying, and interpreting data. In Grade 7, students were expected to choose the appropriate display and justify their choice, and Venn diagrams were introduced. Students also provided convincing arguments based on an analysis of data. In Grade 8, students chose the appropriate measure of center and spread for a data set and justified their choice. They drew conclusions and made predictions by analyzing trends in scatterplots. Students added box-and-whisker plots and histograms to their repertoire of graphical representations. Students in Grade 8 were introduced to the concepts of sampling methods and their effects on validity, and they explored misuses of statistical information. There were no additional statistical learning expectations in Algebra I, Algebra II, and Geometry.

Building on their prior knowledge of statistics, students design and analyze their own statistical studies and in the process become more discerning consumers of statistics in everyday situations. They also develop skills to prepare them for the further use of statistics in their major field of study at the university level.

Learning Expectations Addressed

(DM.4) **Conducting statistical analyses.** The student applies statistical methods to design and conduct a study that addresses one or more particular questions.

The student is expected to:

(A) determine the need for and purpose of a statistical investigation and what type of statistical analysis can be used to answer a specific question or set of questions;

(B) identify the population of interest, select an appropriate sampling technique (such as simple random, stratified, or systematic sampling), and collect data;

(C) identify the variables to be used in a study;

(D) determine possible sources of statistical bias in a study and how such bias may affect the ability to generalize the results;

(E) create displays for a given data set(s) to investigate, compare, and estimate center, shape, spread, and unusual features; and

(F) determine possible sources of variability of data, including sampling, measurement, and induced and natural variability.

(DM.5) **Communicating statistical information.** The student communicates the results of reported and student-generated statistical studies.

The student is expected to:

(A) report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays, and interpreting results in terms of the question studied;

(B) justify the design and the conclusion(s) of statistical studies, including the methods used for each; and

(C) communicate statistical results in both oral and written formats using appropriate statistical and nontechnical language.

Section Overviews

The *Designing a Study* unit is divided into three sections as outlined below. The activities in each section take three to five instructional days.

Section A: Statistical Investigations
Students investigate the purposes of a variety of statistical investigations. They identify the population of interest and the variables to be used in each study. Students then determine the appropriate sampling technique and statistical analysis for each research question.

Section B: Statistical Analyses
Students collect sets of data and create a variety of data displays. They describe the distribution by estimating center, shape, spread, and unusual features. Students also compare and contrast multiple data sets. They then communicate their analyses orally and in writing, using appropriate statistical and nontechnical language.

Section C: Sources of Variability
Students build on the skills and information gathered during the first two sections of this unit to investigate possible sources of variability in the data, including sampling, measurement, and induced and natural variability. They search for possible sources of statistical bias and the effect of such bias on the generalizability of results.

This unit ends with a culminating project that requires students to design and implement a study of interest. They justify the design and execution of the study as well as the conclusion(s) reached. Students communicate their findings in oral, graphical, and written form, using appropriate statistical and nontechnical language.

Unit V: A Discrete Look at Change (3 weeks)

Mathematics Overview

This unit focuses on analyzing data and finding rules to model the data. By looking at recursive models, students expand their set of tools for data analysis.

In Algebra I and Algebra II, students learned about various function families, including linear and exponential functions. This unit builds on students' knowledge of these functions and focuses on recursive rules that model data exhibiting exponential and linear patterns. In this way, students reinforce their understanding of the concepts associated with linear and exponential functions while building a new way to think about modeling these types of data. By introducing a leveling-off value, exponential growth can be extended to the study of logistic growth patterns.

Students add a new function to their library of functions in analyzing cyclical data. The sine function is developed through explorations of data that exhibit periodic behavior and through investigations of the concept of a wrapping function. More work with the sine function follows in a later unit.

Learning Expectations Addressed

(DM.6) **Modeling data.** The student models data, makes predictions, and judges the validity of a prediction.

The student is expected to:

(A) determine whether or not there is a linear relationship in a set of bivariate data by finding the correlation coefficient for the data, and interpret the coefficient as a measure of the strength and direction of the linear relationship; and

(B) collect or use numerical bivariate data to create a scatterplot; select a function (such as linear, exponential, logistic, or trigonometric) to construct a model using the data, justify the selection, and use the model to make predictions.

(DM.7) **Modeling change and relationships**. The student uses mathematical models to represent, analyze, and solve problems involving change.

The student is expected to:

(D) solve problems (such as those involving pattern identification, population growth and decline, and compound interest) using recursion or iteration.

Section Overviews

The *A Discrete Look at Change* unit is divided into three sections as outlined below. The activities in each section take three to five instructional days.

Section A: Linear Models
Students analyze data that follow a linear pattern using recursively defined rules. They make connections between the recursive rule and the standard closed form of the linear rule. Students analyze the finite first differences to determine that a linear function is a good model for these data. They analyze the correlation coefficient of the data to determine the strength of the linear model.

Section B: Exponential and Logistic Models
Students explore data that follow an exponential pattern using the idea of a common ratio between consecutive values. They find recursive rules to model the data and make connections between the recursive rule and the closed form of the exponential rule. A maximum value is introduced to set up the exploration of logistic models. Students explore logistic models by looking at a difference equation.

Section C: Cyclical Models
Students use a model to define a wrapping function and develop the sine function. They analyze the characteristics, including period and amplitude, of the sine function and use the function to model simple cyclical data.

Unit VI: More Models of Change (3 weeks)

Mathematics Overview

This unit focuses on analyzing data and finding mathematical functions (rules) to model real-world data and contexts with continuous functions. After reviewing the modeling process employed with discrete models of change in a previous unit, students expand their set of tools for data analysis by using continuous and piecewise-defined functions encountered in previous courses.

In Algebra I and Algebra II, students learned about a variety of functions, including linear, quadratic, exponential, rational, and step functions. In this unit, students work with context-based rules for business and natural situations to create models. They test these models against data and common sense. In this way, students enhance their ability to understand and use the power and limitations of modeling, to use continuous functions, and to use data and modeling to understand the world in which they live.

In this unit, students use the cyclical functions introduced in an earlier unit to model inherently cyclical phenomena in business and nature. Students also use piecewise- defined functions that are more complicated than the step functions they worked with in earlier courses.

Learning Expectations Addressed

(DM.6) **Modeling data.** The student models data, makes predictions, and judges the validity of a prediction.

The student is expected to:

(A) determine whether or not there is a linear relationship in a set of bivariate data by finding the correlation coefficient for the data and interpret the coefficient as a measure of the strength and direction of the linear relationship; and

(B) collect or use numerical bivariate data to create a scatterplot; select a function (such as linear, exponential, logistic, or trigonometric) to construct a model using the data, justify the selection, and use the model to make predictions.

(DM.7) **Modeling change and relationships**. The student uses mathematical models to represent, analyze, and solve problems involving change.

The student is expected to:

(A) determine or analyze an appropriate growth or decay model (including linear, exponential, and logistic functions) to solve problems (such as those involving inflation, medication dosage, climate change, or bone decay);

(B) determine or analyze an appropriate cyclical model (including trigonometric and other periodic functions) to solve problems (such as those involving phases of the moon, ocean tides, musical tones, or sound);

(C) determine or analyze an appropriate piecewise model to solve problems (such as those involving U.S. tax brackets, cab fare, and postal/shipping rates);

Section Overviews

The *More Models of Change* unit is divided into four sections as outlined below. The activities in each section take three to five instructional days.

Section A: Review of Recursive Function Models and Their Associated Continuous Models
Students review discrete modeling with recursively defined functions and extend their modeling capability to Newton's Law of Cooling. They interpret the statement, "The rate of change of the temperature of a liquid in a room is proportional to the difference between the room's ambient temperature and the liquid's temperature." Students also work with the principle that the rate of change of the amount of a substance is proportional to the amount of the substance.

Section B: Models of Cyclical Phenomena
Students use cyclical functions based on the sine curve to model business cycles in resort towns and cyclical physical phenomena. These models are based on an understanding of business contexts and physical principles. Students determine parameters for a generalized sine curve that will appropriately model several situations in which the period, amplitude, and phase shift vary. Students check their models against existing data.

Section C: Step and Piecewise Models
Students investigate and model the amount of an infection-fighting drug in the bloodstream in repeated applications of the drug at a specific dosage. They also determine the maximum amount of the drug in the bloodstream and the number of repeated applications of the drug needed to attain that maximum amount.

Section D: Making Decisions and the Strength of a Model
Students use scatterplots to assess the validity of a model. They discuss the various types of limitations that occur in models, especially problems with extrapolating outside the data with models that fit the data but do not adhere to business principles or natural laws.

Unit VII: Spatial and Geometric Modeling (3 weeks)

Mathematics Overview

Students apply tools to model geometric situations and solve problems. Vectors and matrices are used as a way to organize and describe situations. Students also extend their knowledge of right triangle trigonometry to include the Law of Sines and the Law of Cosines.

The study of vectors focuses on the definition of vectors. No vector operations, other than scalar multiplication, are used. Vectors are employed to describe and model situations so that students can solve problems. After the concept of a vector is introduced, students continue to use vectors in the remainder of the unit to model problems.

In geometry, students learned about right triangle trigonometry. In this unit, they use that knowledge to solve problems involving inaccessible distances. Many problems that students solve are modeled using vectors. Students then apply new tools, such as the Law of Sines or the Law of Cosines, to solve problems.

Matrices are explored as a way to organize information. In Algebra II, students were introduced to matrices and some matrix operations. This unit builds on that foundation and looks at ways that matrices can be used to represent geometric transformations.

Learning Expectations Addressed

(DM.8) **Modeling with geometric tools**. The student uses a variety of tools and methods to represent and solve problems involving static and dynamic situations.

The student is expected to:

(A) create and use two- and three-dimensional representations of authentic situations using paper techniques or dynamic geometric environments for computer-aided design and other applications;

(B) solve problems and represent situations using vectors in areas such as transportation, computer graphics, and the physics of force and motion;

(C) solve problems and represent geometric transformations using matrices in fields such as computer animations; and

(D) solve geometric problems involving inaccessible distances (such as those encountered when building a bridge, constructing a skyscraper, or mapping planetary distances) using basic trigonometric principles.

Section Overviews

The *Spatial and Geometric Modeling* unit is divided into three sections as outlined below. The activities in each section take three to five instructional days.

Section A: Vectors
Students define a vector as having magnitude and direction. They understand scalar multiplication of vectors and use vectors to model situations including physical force and motion as well as computer animations.

Section B: Inaccessible Distances
Students model situations geometrically using vectors and then apply trigonometric principles to solve problems involving distances that cannot be easily measured. Students use their understanding of right triangle trigonometry. They also explore and apply new trigonometric rules, such as the Law of Sines and the Law of Cosines.

Section C: Transformations
Students use their knowledge of vectors to describe dilations and translations in a plane. They then apply their knowledge of matrices from prior courses to think about matrices in a different way. Matrices are used as organizers of information. Students also investigate matrices that can be used to describe geometric transformations, a concept that brings together many other concepts, including vectors, trigonometry, geometric transformations, and matrix operations. Matrices can be employed to describe translations, dilations, rotations, and reflections.

Unit VIII: Networks and Graphs (3 weeks)

Mathematics Overview

This unit focuses on the creation of models that represent real-world contexts involving networks and graphs and the use of these networks and graphs to investigate real-world scheduling and biological sequencing problems. Using networks and graphs, students extend their ability to solve abstract and concrete problems.

Although networks and graphs have geometrical connections (in that they are drawn in two dimensions with points, lines, and curves), the mathematical reasoning required to create, understand, and use networks and graphs is new to most students.

Learning Expectations Addressed

(DM.9) **Network modeling for decision making.** The student uses a variety of network models represented graphically to organize data in quantitative situations, make informed decisions, and solve problems.

The student is expected to:

(A) solve problems involving situations (such as scheduling tasks, making deliveries, and finding shortest routes) that can be represented by a vertex-edge graph, and find critical paths, Euler paths, or minimal spanning trees; and

(B) construct, analyze, and interpret flow charts in order to develop an algorithm to describe a particular process (such as designing quality control procedures for a manufacturing facility).

Section Overviews

The *Networks and Graphs* unit is divided into five sections as outlined below. The activities in each section take two to four instructional days.

Section A: Circuits and Paths
Students use graphs and the definitions of circuits and paths to study a situation like the Konigsberg bridge problem to determine if certain conditions can be satisfied. They use theorems and algorithms to solve such problems.

Section B: Spanning Trees
Students represent situations with tree diagrams and then look at ways of determining the spanning trees that solve questions arising from the situation. The problem in biological sequencing and evolutionary tree construction is investigated. Some algorithms for finding spanning trees are presented and used without proof.

Section C: Graph Colorings
Students consider problems that can be resolved by coloring graphs. They create graphs from a description of a situation and then determine if the graphs can be colored in specific ways using theorems and algorithms.

Section D: Graph Structure
Students create graph structures to use in determining the best methods for scheduling tasks and making assignments. These structures are more complicated than the structures encountered in the first two sections of this unit and are more generally applicable.

Section E: PERT Charts
Students study the scheduling of projects using the Program Evaluation and Review Technique (PERT). They work with information about a project, including tasks and their time constraints along with interrelationships between and among tasks. Optionally, freely available PERT software is employed to determine the scheduling pattern for the project.

Unit IX: Decision Making in Finance (2 weeks)

Mathematics Overview

This unit focuses on the financial decisions that surround borrowing, loaning, and investing money and how the time value of money affects such decisions.

In the *More Models of Change* unit, students studied the basic mathematical structure involved in such decision making: $f(t) = ab^t$, the general exponential function. Students use this function as the basis for more complex functions that model change in a variety of financial situations.

Learning Expectations Addressed

(DM.10) **Mathematical decision making in finance**. The student creates and analyzes mathematical models to make decisions related to earning, investing, spending, and borrowing money.

The student is expected to:

(A) determine, represent, and analyze mathematical models for various types of income (such as commission, salary, and hourly wage) to determine the best option for a given situation;

(B) determine, represent, and analyze mathematical models for expenditures (such as credit cards, auto financing, cell phone plans, and financial aid) to determine the best option for a given situation; and

(C) determine, represent, and analyze mathematical models and appropriate representations (such as expected values or probability distributions) for various types of loans and investments (such as savings plans and real estate) to determine the best loan or investment plan for a given situation.

Section Overviews

The *Decision Making in Finance* unit is divided into five sections as outlined below. The activities in each section take two to three instructional days. A graphing calculator or other technology is required in this unit.

<u>Section A: Future Value of an Investment</u>
Students study the future value of an investment. The closed-form exponential formula for the future value of an investment with interest compounded annually and monthly is developed. Students also compare and contrast the nominal interest rate with the annual percentage rate (APR). After completing this section, students can answer questions about the difference between simple and compound interest in given situations.

<u>Section B: Present Value of an Investment</u>
Students develop the formula for the present value of an investment from the formula for the future value of an investment developed in the first section. After completing this section, students can compare investment scenarios using the concept of present value of an investment.

Section C: Annuities
Students develop and study the formula for determining the future and present value of an annuity. They also study the effect of increasing the interest rate on the present value of an annuity. Finally, students study perpetuities. After completing this section, students can solve problems involving annuities.

Section D: Credit Card Debt
Students study different real-world credit card offers. They investigate the formula for determining the monthly payment to retire a debt at a fixed rate in a fixed number of monthly payments. Students compare and contrast different credit card offers. They also study credit card statements to fully understand the concept of minimum payment, the length of time to pay off a credit card debt using the minimum payment, and the APR of such minimum payments. After completing this section, students can solve problems involving credit card debt.

Section E: Comparing Automobile Loan Proposals (optional)
Students study methods of comparing car loans using the tools developed in the four previous sections. They compare and contrast the bank or credit union loans, automobile dealer loans, and car purchases that have a cash-back feature. After completing this section, students can make informed financial decisions when purchasing a vehicle.

AMDM Supplemental Unit (to be developed in Fall 2009)

Unit X: Decision Making in Voting and Selection Processes (Suggested 2 weeks)

Mathematics Overview

This unit allows students to learn, use, and analyze numerical methods in decision making as it appears in a range of selection and voting contexts. By working with familiar mathematical tools, students improve their ability to make informed decisions—especially in everyday situations, such as those where television viewers select a winner or where students design a process for voting in a school election.

Learning Expectations Addressed

(DM.11) **Mathematical decision making in voting and selection**. The student analyzes and evaluates the mathematics behind various methods of voting and selection.

The student is expected to:

(A) evaluate various voting and selection processes to determine an appropriate method for a given situation; and [combines A and C from previous version]

(B) apply various ranking algorithms (such as methods used to compute class rank or athletic team rankings) to determine an appropriate method for a given situation.

Section Overviews

The *Decision Making in Voting and Selection Processes* unit is divided into three sections as outlined below. The activities in each section take three to five instructional days.

Section A, "Cutting the Cake": Division Techniques
Students use various mathematical methods such as taking turns, divide and choose, and so forth in situations requiring the division of objects among several people such that each person receives his/her share. The classic cake-cutting problem is used to illustrate methods that are then applied to situations such as division of property, resource allocation, and the Electoral College.

Section B, "Who's #1?": Using Ranking Methods
Students apply various ranking algorithms in situations involving school class rank, athletic team rankings used to determine postseason eligibility and position in drafting new players, multiweek TV contest winners, and ladder tournaments. Students also analyze the flaws in some of these methods.

Section C, "And the Winner is …": Using Voting Methods
Students apply and evaluate various selection processes such as majority rule, plurality voting, approval voting, and pairwise voting used to determine the winner in an election. They also examine ways that such methods can be inappropriately manipulated to produce results that may not be representative.