

## Text of Adopted Amendments to 19 TAC

### Chapter 111. Texas Essential Knowledge and Skills for Mathematics

#### Subchapter A. Elementary

##### §111.11. Implementation of Texas Essential Knowledge and Skills for Mathematics, Grades K-5.

The provisions of this subchapter shall be implemented by school districts beginning with the 2006-2007 school year.

##### §111.12. Mathematics, Kindergarten.

(a) Introduction.

- (1) Within a well-balanced mathematics curriculum, the primary focal points at Kindergarten are developing whole-number concepts and using patterns and sorting to explore number, data, and shape.
- (2) Throughout mathematics in Kindergarten-Grade 2, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use numbers in ordering, labeling, and expressing quantities and relationships to solve problems and translate informal language into mathematical language and symbols. Students use objects to create and identify patterns and use those patterns to express relationships, make predictions, and solve problems as they build an understanding of number, operation, shape, and space. Students progress from informal to formal language to describe two- and three-dimensional geometric figures and likenesses in the physical world. Students begin to develop measurement concepts as they identify and compare attributes of objects and situations. Students collect, organize, and display data and use information from graphs to answer questions, make summary statements, and make informal predictions based on their experiences.
- (3) Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.
- (4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Kindergarten-Grade 2, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

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(b) Knowledge and skills.

(K.1) **Number, operation, and quantitative reasoning.** The student uses numbers to name quantities.

The student is expected to:

- (A) use one-to-one correspondence and language such as more than, same number as, or two less than to describe relative sizes of sets of concrete objects;

- (K.2) **Number, operation, and quantitative reasoning.** The student describes order of events or objects.
- (K.3) **Number, operation, and quantitative reasoning.** The student recognizes that there are quantities less than a whole.
- (K.4) **Number, operation, and quantitative reasoning.** The student models addition (joining) and subtraction (separating).
- (K.5) **Patterns, relationships, and algebraic thinking.** The student identifies, extends, and creates patterns.
- (K.6) **Patterns, relationships, and algebraic thinking.** The student uses patterns to make predictions.
- (K.7) **Geometry and spatial reasoning.** The student describes the relative positions of objects.
- (K.8) **Geometry and spatial reasoning.** The student uses attributes to determine how objects are alike and different.
- (B) use sets of concrete objects to represent quantities given in verbal or written form (through 20); and
- (C) use numbers to describe how many objects are in a set (through 20) using verbal and symbolic descriptions.
- The student is expected to:
- (A) use language such as before or after to describe relative position in a sequence of events or objects; and
- (B) name the ordinal positions in a sequence such as first, second, third, etc.
- The student is expected to:
- (A) share a whole by separating it into two equal parts; and
- (B) explain why a given part is half of the whole.
- The student is expected to model and create addition and subtraction problems in real situations with concrete objects.
- The student is expected to identify, extend, and create patterns of sounds, physical movement, and concrete objects.
- The student is expected to:
- (A) use patterns to predict what comes next, including cause-and-effect relationships; and
- (B) count by ones to 100.
- The student is expected to:
- (A) describe one object in relation to another using informal language such as over, under, above, and below; and
- (B) place an object in a specified position.
- The student is expected to:
- (A) describe and identify an object by its attributes using informal language;
- (B) compare two objects based on their attributes; and

(K.9) **Geometry and spatial reasoning.** The student recognizes attributes of two- and three-dimensional geometric figures.

- (C) sort a variety of objects including two- and three-dimensional geometric figures according to their attributes and describe how the objects are sorted.

The student is expected to:

- (A) describe and compare the attributes of real-life objects such as balls, boxes, cans, and cones or models of three-dimensional geometric figures;
- (B) recognize shapes in real-life three-dimensional geometric figures or models of three-dimensional geometric figures; and
- (C) describe, identify, and compare circles, triangles, rectangles, and squares (a special type of rectangle).

(K.10) **Measurement.** The student directly compares the attributes of length, area, weight/mass, capacity, and/or relative temperature. The student uses comparative language to solve problems and answer questions.

The student is expected to:

- (A) compare and order two or three concrete objects according to length (longer/shorter than, or the same);
- (B) compare the areas of two flat surfaces of two-dimensional figures (covers more, covers less, or covers the same);
- (C) compare two containers according to capacity (holds more, holds less, or holds the same);
- (D) compare two objects according to weight/mass (heavier than, lighter than or equal to); and
- (E) compare situations or objects according to relative temperature (hotter/colder than, or the same as).

(K.11) **Measurement.** The student uses time to describe, compare, and order events and situations.

The student is expected to:

- (A) compare events according to duration such as more time than or less time than;
- (B) sequence events (up to three); and
- (C) read a calendar using days, weeks, and months.

(K.12) **Probability and statistics.** The student constructs and uses graphs of real objects or pictures to answer questions.

The student is expected to:

- (A) construct graphs using real objects or pictures in order to answer questions; and
- (B) use information from a graph of real objects or pictures in order to answer questions.

(K.13) **Underlying processes and mathematical tools.** The student applies Kindergarten mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:

- (A) identify mathematics in everyday situations;
- (B) solve problems with guidance that incorporates the processes of understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
- (C) select or develop an appropriate problem-solving strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem; and
- (D) use tools such as real objects, manipulatives, and technology to solve problems.

(K.14) **Underlying processes and mathematical tools.** The student communicates about Kindergarten mathematics using informal language.

The student is expected to:

- (A) communicate mathematical ideas using objects, words, pictures, numbers, and technology; and
- (B) relate everyday language to mathematical language and symbols.

(K.15) **Underlying processes and mathematical tools.** The student uses logical reasoning.

The student is expected to justify his or her thinking using objects, words, pictures, numbers, and technology.

### **§111.13. Mathematics, Grade 1.**

(a) Introduction.

- (1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 1 are building number sense through number relationships, adding and subtracting whole numbers, organizing and analyzing data, and working with two- and three-dimensional geometric figures.
- (2) Throughout mathematics in Kindergarten-Grade 2, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use numbers in ordering, labeling, and expressing quantities and relationships to solve problems and translate informal language into mathematical language and symbols. Students use objects to create and identify patterns and use those patterns to express relationships, make predictions, and solve problems as they build an understanding of number, operation, shape, and space. Students progress from informal to formal language to describe two- and three-dimensional geometric figures and likenesses in the physical world. Students begin to develop measurement concepts as they identify and compare attributes of objects and situations. Students collect, organize, and display data and use information from graphs to answer questions, make summary statements, and make informal predictions based on their experiences.

- (3) Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.
- (4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Kindergarten-Grade 2, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

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(b) Knowledge and skills.

- (1.1) **Number, operation, and quantitative reasoning.** The student uses whole numbers to describe and compare quantities.

The student is expected to:

- (A) compare and order whole numbers up to 99 (less than, greater than, or equal to) using sets of concrete objects and pictorial models;
- (B) create sets of tens and ones using concrete objects to describe, compare, and order whole numbers;
- (C) identify individual coins by name and value and describe relationships among them; and
- (D) read and write numbers to 99 to describe sets of concrete objects.

- (1.2) **Number, operation, and quantitative reasoning.** The student uses pairs of whole numbers to describe fractional parts of whole objects or sets of objects.

The student is expected to:

- (A) separate a whole into two, three, or four equal parts and use appropriate language to describe the parts such as three out of four equal parts; and
- (B) use appropriate language to describe part of a set such as three out of the eight crayons are red.

- (1.3) **Number, operation, and quantitative reasoning.** The student recognizes and solves problems in addition and subtraction situations.

The student is expected to:

- (A) model and create addition and subtraction problem situations with concrete objects and write corresponding number sentences; and
- (B) use concrete and pictorial models to apply basic addition and subtraction facts (up to  $9 + 9 = 18$  and  $18 - 9 = 9$ ).

- (1.4) **Patterns, relationships, and algebraic thinking.** The student uses repeating patterns and additive patterns to make predictions.

The student is expected to identify, describe, and extend concrete and pictorial patterns in order to make predictions and solve problems.

- (1.5) **Patterns, relationships, and algebraic thinking.** The student recognizes patterns in numbers and operations.

The student is expected to:

- (A) use patterns to skip count by twos, fives, and tens;
- (B) find patterns in numbers, including odd and even;
- (C) compare and order whole numbers using place value;
- (D) use patterns to develop strategies to solve basic addition and basic subtraction problems; and
- (E) identify patterns in related addition and subtraction sentences (fact families for sums to 18) such as  $2 + 3 = 5$ ,  $3 + 2 = 5$ ,  $5 - 2 = 3$ , and  $5 - 3 = 2$ .

- (1.6) **Geometry and spatial reasoning.** The student uses attributes to identify two- and three-dimensional geometric figures. The student compares and contrasts two- and three-dimensional geometric figures or both.

The student is expected to:

- (A) describe and identify two-dimensional geometric figures, including circles, triangles, rectangles, and squares (a special type of rectangle);
- (B) describe and identify three-dimensional geometric figures, including spheres, rectangular prisms (including cubes), cylinders, and cones;
- (C) describe and identify two- and three-dimensional geometric figures in order to sort them according to a given attribute using informal and formal language; and
- (D) use concrete models to combine two-dimensional geometric figures to make new geometric figures.

- (1.7) **Measurement.** The student directly compares the attributes of length, area, weight/mass, capacity, and temperature. The student uses comparative language to solve problems and answer questions. The student selects and uses nonstandard units to describe length.

The student is expected to:

- (A) estimate and measure length using nonstandard units such as paper clips or sides of color tiles;
- (B) compare and order two or more concrete objects according to length (from longest to shortest);
- (C) describe the relationship between the size of the unit and the number of units needed to measure the length of an object;
- (D) compare and order the area of two or more two-dimensional surfaces (from covers the most to covers the least);

- (E) compare and order two or more containers according to capacity (from holds the most to holds the least);
- (F) compare and order two or more objects according to weight/mass (from heaviest to lightest); and
- (G) compare and order two or more objects according to relative temperature (from hottest to coldest).
- (1.8) **Measurement.** The student understands that time can be measured. The student uses time to describe and compare situations.
- The student is expected to:
- (A) order three or more events according to duration; and
- (B) read time to the hour and half-hour using analog and digital clocks.
- (1.9) **Probability and statistics.** The student displays data in an organized form.
- The student is expected to:
- (A) collect and sort data; and
- (B) use organized data to construct real-object graphs, picture graphs, and bar-type graphs.
- (1.10) **Probability and statistics.** The student uses information from organized data.
- The student is expected to:
- (A) draw conclusions and answer questions using information organized in real-object graphs, picture graphs, and bar-type graphs; and
- (B) identify events as certain or impossible such as drawing a red crayon from a bag of green crayons.
- (1.11) **Underlying processes and mathematical tools.** The student applies Grade 1 mathematics to solve problems connected to everyday experiences and activities in and outside of school.
- The student is expected to:
- (A) identify mathematics in everyday situations;
- (B) solve problems with guidance that incorporates the processes of understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
- (C) select or develop an appropriate problem-solving plan or strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem; and
- (D) use tools such as real objects, manipulatives, and technology to solve problems.

(1.12) **Underlying processes and mathematical tools.** The student communicates about Grade 1 mathematics using informal language.

The student is expected to:

- (A) explain and record observations using objects, words, pictures, numbers, and technology; and
- (B) relate informal language to mathematical language and symbols.

(1.13) **Underlying processes and mathematical tools.** The student uses logical reasoning.

The student is expected to justify his or her thinking using objects, words, pictures, numbers, and technology.

### **§111.14. Mathematics, Grade 2.**

(a) Introduction.

- (1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 2 are developing an understanding of the base-ten place value system, comparing and ordering whole numbers, applying addition and subtraction, and using measurement processes.
- (2) Throughout mathematics in Kindergarten-Grade 2, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use numbers in ordering, labeling, and expressing quantities and relationships to solve problems and translate informal language into mathematical language and symbols. Students use objects to create and identify patterns and use those patterns to express relationships, make predictions, and solve problems as they build an understanding of number, operation, shape, and space. Students progress from informal to formal language to describe two- and three-dimensional geometric figures and likenesses in the physical world. Students begin to develop measurement concepts as they identify and compare attributes of objects and situations. Students collect, organize, and display data and use information from graphs to answer questions, make summary statements, and make informal predictions based on their experiences.
- (3) Throughout mathematics in Kindergarten-Grade 2, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Kindergarten-Grade 2 use basic number sense to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 2, students know basic addition and subtraction facts and are using them to work flexibly, efficiently, and accurately with numbers during addition and subtraction computation.
- (4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Kindergarten-Grade 2, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

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(b) Knowledge and skills.

(2.1) **Number, operation, and quantitative reasoning.** The student understands how place value is used to represent whole numbers.

The student is expected to:

- (A) use concrete models of hundreds, tens, and ones to represent a given whole number (up to 999) in various ways;
- (B) use place value to read, write, and describe the value of whole numbers to 999; and

- (2.2) **Number, operation, and quantitative reasoning.** The student describes how fractions are used to name parts of whole objects or sets of objects.
- (2.3) **Number, operation, and quantitative reasoning.** The student adds and subtracts whole numbers to solve problems.
- (2.4) **Number, operation, and quantitative reasoning.** The student models multiplication and division.
- (2.5) **Patterns, relationships, and algebraic thinking.** The student uses patterns in numbers and operations.
- (C) use place value to compare and order whole numbers to 999 and record the comparisons using numbers and symbols (<, =, >).
- The student is expected to:
- (A) use concrete models to represent and name fractional parts of a whole object (with denominators of 12 or less);
- (B) use concrete models to represent and name fractional parts of a set of objects (with denominators of 12 or less); and
- (C) use concrete models to determine if a fractional part of a whole is closer to 0,  $\frac{1}{2}$ , or 1.
- The student is expected to:
- (A) recall and apply basic addition and subtraction facts (to 18);
- (B) model addition and subtraction of two-digit numbers with objects, pictures, words, and numbers;
- (C) select addition or subtraction to solve problems using two-digit numbers, whether or not regrouping is necessary;
- (D) determine the value of a collection of coins up to one dollar; and
- (E) describe how the cent symbol, dollar symbol, and the decimal point are used to name the value of a collection of coins.
- The student is expected to:
- (A) model, create, and describe multiplication situations in which equivalent sets of concrete objects are joined; and
- (B) model, create, and describe division situations in which a set of concrete objects is separated into equivalent sets.
- The student is expected to:
- (A) find patterns in numbers such as in a 100s chart;
- (B) use patterns in place value to compare and order whole numbers through 999; and

- (2.6) **Patterns, relationships, and algebraic thinking.** The student uses patterns to describe relationships and make predictions.
- (2.7) **Geometry and spatial reasoning.** The student uses attributes to identify two- and three-dimensional geometric figures. The student compares and contrasts two- and three-dimensional geometric figures or both.
- (2.8) **Geometry and spatial reasoning.** The student recognizes that a line can be used to represent a set of numbers and its properties.
- (2.9) **Measurement.** The student directly compares the attributes of length, area, weight/mass, and capacity, and uses comparative language to solve problems and answer questions. The student selects and uses nonstandard units to describe length, area, capacity, and weight/mass. The student recognizes and uses models that approximate standard units (from both SI, also known as metric, and customary systems) of length, weight/mass, capacity, and time.
- (C) use patterns and relationships to develop strategies to remember basic addition and subtraction facts. Determine patterns in related addition and subtraction number sentences (including fact families) such as  $8 + 9 = 17$ ,  $9 + 8 = 17$ ,  $17 - 8 = 9$ , and  $17 - 9 = 8$ .
- The student is expected to:
- (A) generate a list of paired numbers based on a real-life situation such as number of tricycles related to number of wheels;
- (B) identify patterns in a list of related number pairs based on a real-life situation and extend the list; and
- (C) identify, describe, and extend repeating and additive patterns to make predictions and solve problems.
- The student is expected to:
- (A) describe attributes (the number of vertices, faces, edges, sides) of two- and three-dimensional geometric figures such as circles, polygons, spheres, cones, cylinders, prisms, and pyramids, etc.;
- (B) use attributes to describe how 2 two-dimensional figures or 2 three-dimensional geometric figures are alike or different; and
- (C) cut two-dimensional geometric figures apart and identify the new geometric figures formed.
- The student is expected to use whole numbers to locate and name points on a number line.
- The student is expected to:
- (A) identify concrete models that approximate standard units of length and use them to measure length;
- (B) select a non-standard unit of measure such as square tiles to determine the area of a two-dimensional surface;
- (C) select a non-standard unit of measure such as a bathroom cup or a jar to determine the capacity of a given container; and
- (D) select a non-standard unit of measure such as beans or marbles to determine the weight/mass of a given object.

(2.10) **Measurement.** The student uses standard tools to estimate and measure time and temperature (in degrees Fahrenheit).

The student is expected to:

- (A) read a thermometer to gather data;
- (B) read and write times shown on analog and digital clocks using five-minute increments; and
- (C) describe activities that take approximately one second, one minute, and one hour.

(2.11) **Probability and statistics.** The student organizes data to make it useful for interpreting information.

The student is expected to:

- (A) construct picture graphs and bar-type graphs;
- (B) draw conclusions and answer questions based on picture graphs and bar-type graphs; and
- (C) use data to describe events as more likely or less likely such as drawing a certain color crayon from a bag of seven red crayons and three green crayons.

(2.12) **Underlying processes and mathematical tools.** The student applies Grade 2 mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:

- (A) identify the mathematics in everyday situations;
- (B) solve problems with guidance that incorporates the processes of understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
- (C) select or develop an appropriate problem-solving plan or strategy including drawing a picture, looking for a pattern, systematic guessing and checking, or acting it out in order to solve a problem; and
- (D) use tools such as real objects, manipulatives, and technology to solve problems.

(2.13) **Underlying processes and mathematical tools.** The student communicates about Grade 2 mathematics using informal language.

The student is expected to:

- (A) explain and record observations using objects, words, pictures, numbers, and technology; and
- (B) relate informal language to mathematical language and symbols.

(2.14) **Underlying processes and mathematical tools.** The student uses logical reasoning.

The student is expected to justify his or her thinking using objects, words, pictures, numbers, and technology.

### §111.15. Mathematics, Grade 3.

(a) Introduction.

- (1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 3 are multiplying and dividing whole numbers, connecting fraction symbols to fractional quantities, and standardizing language and procedures in geometry and measurement.
- (2) Throughout mathematics in Grades 3-5, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use algorithms for addition, subtraction, multiplication, and division as generalizations connected to concrete experiences; and they concretely develop basic concepts of fractions and decimals. Students use appropriate language and organizational structures such as tables and charts to represent and communicate relationships, make predictions, and solve problems. Students select and use formal language to describe their reasoning as they identify, compare, and classify two- or three-dimensional geometric figures; and they use numbers, standard units, and measurement tools to describe and compare objects, make estimates, and solve application problems. Students organize data, choose an appropriate method to display the data, and interpret the data to make decisions and predictions and solve problems.
- (3) Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division computation.
- (4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 3-5, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

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(b) Knowledge and skills.

(3.1) **Number, operation, and quantitative reasoning.** The student uses place value to communicate about increasingly large whole numbers in verbal and written form, including money.

The student is expected to:

- (A) use place value to read, write (in symbols and words), and describe the value of whole numbers through 999,999;
- (B) use place value to compare and order whole numbers through 9,999; and
- (C) determine the value of a collection of coins and bills.

(3.2) **Number, operation, and quantitative reasoning.** The student uses fraction names and symbols (with denominators of 12 or less) to describe fractional parts of whole objects or sets of objects.

The student is expected to:

- (A) construct concrete models of fractions;
- (B) compare fractional parts of whole objects or sets of objects in a problem situation using concrete models;

- (3.3) **Number, operation, and quantitative reasoning.** The student adds and subtracts to solve meaningful problems involving whole numbers.
- (3.4) **Number, operation, and quantitative reasoning.** The student recognizes and solves problems in multiplication and division situations.
- (3.5) **Number, operation, and quantitative reasoning.** The student estimates to determine reasonable results.
- (3.6) **Patterns, relationships, and algebraic thinking.** The student uses patterns to solve problems.
- (3.7) **Patterns, relationships, and algebraic thinking.** The student uses lists, tables, and charts to express patterns and relationships.
- (C) use fraction names and symbols to describe fractional parts of whole objects or sets of objects; and
- (D) construct concrete models of equivalent fractions for fractional parts of whole objects.
- The student is expected to:
- (A) model addition and subtraction using pictures, words, and numbers; and
- (B) select addition or subtraction and use the operation to solve problems involving whole numbers through 999.
- The student is expected to:
- (A) learn and apply multiplication facts through 12 by 12 using concrete models and objects;
- (B) solve and record multiplication problems (up to two digits times one digit); and
- (C) use models to solve division problems and use number sentences to record the solutions.
- The student is expected to:
- (A) round whole numbers to the nearest ten or hundred to approximate reasonable results in problem situations; and
- (B) use strategies including rounding and compatible numbers to estimate solutions to addition and subtraction problems.
- The student is expected to:
- (A) identify and extend whole-number and geometric patterns to make predictions and solve problems;
- (B) identify patterns in multiplication facts using concrete objects, pictorial models, or technology; and
- (C) identify patterns in related multiplication and division sentences (fact families) such as  $2 \times 3 = 6$ ,  $3 \times 2 = 6$ ,  $6 \div 2 = 3$ ,  $6 \div 3 = 2$ .
- The student is expected to:
- (A) generate a table of paired numbers based on a real-life situation such as insects and legs; and

- (3.8) **Geometry and spatial reasoning.**  
The student uses formal geometric vocabulary.
- (3.9) **Geometry and spatial reasoning.**  
The student recognizes congruence and symmetry.
- (3.10) **Geometry and spatial reasoning.**  
The student recognizes that a line can be used to represent numbers and fractions and their properties and relationships.
- (3.11) **Measurement.** The student directly compares the attributes of length, area, weight/mass, and capacity, and uses comparative language to solve problems and answer questions. The student selects and uses standard units to describe length, area, capacity/volume, and weight/mass.
- (3.12) **Measurement.** The student reads and writes time and measures temperature in degrees Fahrenheit to solve problems.
- (B) identify and describe patterns in a table of related number pairs based on a meaningful problem and extend the table.
- The student is expected to identify, classify, and describe two- and three-dimensional geometric figures by their attributes. The student compares two- dimensional figures, three-dimensional figures, or both by their attributes using formal geometry vocabulary.
- The student is expected to:
- (A) identify congruent two-dimensional figures;
- (B) create two-dimensional figures with lines of symmetry using concrete models and technology; and
- (C) identify lines of symmetry in two-dimensional geometric figures.
- The student is expected to locate and name points on a number line using whole numbers and fractions, including halves and fourths.
- The student is expected to:
- (A) use linear measurement tools to estimate and measure lengths using standard units;
- (B) use standard units to find the perimeter of a shape;
- (C) use concrete and pictorial models of square units to determine the area of two-dimensional surfaces;
- (D) identify concrete models that approximate standard units of weight/mass and use them to measure weight/mass;
- (E) identify concrete models that approximate standard units for capacity and use them to measure capacity; and
- (F) use concrete models that approximate cubic units to determine the volume of a given container or other three-dimensional geometric figure.
- The student is expected to:
- (A) use a thermometer to measure temperature; and

- (3.13) **Probability and statistics.** The student solves problems by collecting, organizing, displaying, and interpreting sets of data.
- (B) tell and write time shown on analog and digital clocks.
- The student is expected to:
- (A) collect, organize, record, and display data in pictographs and bar graphs where each picture or cell might represent more than one piece of data;
- (B) interpret information from pictographs and bar graphs; and
- (C) use data to describe events as more likely than, less likely than, or equally likely as.
- (3.14) **Underlying processes and mathematical tools.** The student applies Grade 3 mathematics to solve problems connected to everyday experiences and activities in and outside of school.
- The student is expected to:
- (A) identify the mathematics in everyday situations;
- (B) solve problems that incorporate understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
- (C) select or develop an appropriate problem-solving plan or strategy, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
- (D) use tools such as real objects, manipulatives, and technology to solve problems.
- (3.15) **Underlying processes and mathematical tools.** The student communicates about Grade 3 mathematics using informal language.
- The student is expected to:
- (A) explain and record observations using objects, words, pictures, numbers, and technology; and
- (B) relate informal language to mathematical language and symbols.
- (3.16) **Underlying processes and mathematical tools.** The student uses logical reasoning.
- The student is expected to:
- (A) make generalizations from patterns or sets of examples and nonexamples; and
- (B) justify why an answer is reasonable and explain the solution process.

#### **§111.16. Mathematics, Grade 4.**

(a) Introduction.

- (1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 4 are comparing and ordering fractions and decimals, applying multiplication and division, and developing ideas related to congruence and symmetry.
- (2) Throughout mathematics in Grades 3-5, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use algorithms for addition, subtraction, multiplication, and division as generalizations connected to concrete experiences; and they concretely develop basic concepts of fractions and decimals. Students use appropriate language and organizational structures such as tables and charts to represent and communicate relationships, make predictions, and solve problems. Students select and use formal language to describe their reasoning as they identify, compare, and classify two- or three-dimensional geometric figures; and they use numbers, standard units, and measurement tools to describe and compare objects, make estimates, and solve application problems. Students organize data, choose an appropriate method to display the data, and interpret the data to make decisions and predictions and solve problems.
- (3) Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division computation.
- (4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 3-5, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

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(b) Knowledge and skills.

(4.1) **Number, operation, and quantitative reasoning.** The student uses place value to represent whole numbers and decimals.

The student is expected to:

- (A) use place value to read, write, compare, and order whole numbers through 999,999,999; and
- (B) use place value to read, write, compare, and order decimals involving tenths and hundredths, including money, using concrete objects and pictorial models.

(4.2) **Number, operation, and quantitative reasoning.** The student describes and compares fractional parts of whole objects or sets of objects.

The student is expected to:

- (A) use concrete objects and pictorial models to generate equivalent fractions;
- (B) model fraction quantities greater than one using concrete objects and pictorial models;
- (C) compare and order fractions using concrete objects and pictorial models; and

- (4.3) **Number, operation, and quantitative reasoning.** The student adds and subtracts to solve meaningful problems involving whole numbers and decimals.
- (4.4) **Number, operation, and quantitative reasoning.** The student multiplies and divides to solve meaningful problems involving whole numbers.
- (4.5) **Number, operation, and quantitative reasoning.** The student estimates to determine reasonable results.
- (4.6) **Patterns, relationships, and algebraic thinking.** The student uses patterns in multiplication and division.
- (4.7) **Patterns, relationships, and algebraic thinking.** The student uses organizational structures to analyze and describe patterns and relationships.
- (D) relate decimals to fractions that name tenths and hundredths using concrete objects and pictorial models.
- The student is expected to:
- (A) use addition and subtraction to solve problems involving whole numbers; and
- (B) add and subtract decimals to the hundredths place using concrete objects and pictorial models.
- The student is expected to:
- (A) model factors and products using arrays and area models;
- (B) represent multiplication and division situations in picture, word, and number form;
- (C) recall and apply multiplication facts through  $12 \times 12$ ;
- (D) use multiplication to solve problems (no more than two digits times two digits without technology); and
- (E) use division to solve problems (no more than one-digit divisors and three-digit dividends without technology).
- The student is expected to:
- (A) round whole numbers to the nearest ten, hundred, or thousand to approximate reasonable results in problem situations; and
- (B) use strategies including rounding and compatible numbers to estimate solutions to multiplication and division problems.
- The student is expected to:
- (A) use patterns and relationships to develop strategies to remember basic multiplication and division facts (such as the patterns in related multiplication and division number sentences (fact families) such as  $9 \times 9 = 81$  and  $81 \div 9 = 9$ ); and
- (B) use patterns to multiply by 10 and 100.
- The student is expected to describe the relationship between two sets of related data such as ordered pairs in a table.

- (4.8) **Geometry and spatial reasoning.**  
The student identifies and describes attributes of geometric figures using formal geometric language.

The student is expected to:

- (A) identify and describe right, acute, and obtuse angles;
- (B) identify and describe parallel and intersecting (including perpendicular) lines using concrete objects and pictorial models; and
- (C) use essential attributes to define two- and three-dimensional geometric figures.

- (4.9) **Geometry and spatial reasoning.**  
The student connects transformations to congruence and symmetry.

The student is expected to:

- (A) demonstrate translations, reflections, and rotations using concrete models;
- (B) use translations, reflections, and rotations to verify that two shapes are congruent; and
- (C) use reflections to verify that a shape has symmetry.

- (4.10) **Geometry and spatial reasoning.**  
The student recognizes the connection between numbers and their properties and points on a line.

The student is expected to locate and name points on a number line using whole numbers, fractions such as halves and fourths, and decimals such as tenths.

- (4.11) **Measurement.** The student applies measurement concepts. The student is expected to estimate and measure to solve problems involving length (including perimeter) and area. The student uses measurement tools to measure capacity/volume and weight/mass.

The student is expected to:

- (A) estimate and use measurement tools to determine length (including perimeter), area, capacity and weight/mass using standard units SI (metric) and customary;
- (B) perform simple conversions between different units of length, between different units of capacity, and between different units of weight within the customary measurement system;
- (C) use concrete models of standard cubic units to measure volume;
- (D) estimate volume in cubic units; and
- (E) explain the difference between weight and mass.

- (4.12) **Measurement.** The student applies measurement concepts. The student measures time and temperature (in degrees Fahrenheit and Celsius).

The student is expected to:

- (A) use a thermometer to measure temperature and changes in temperature; and

- (4.13) **Probability and statistics.** The student solves problems by collecting, organizing, displaying, and interpreting sets of data.
- (4.14) **Underlying processes and mathematical tools.** The student applies Grade 4 mathematics to solve problems connected to everyday experiences and activities in and outside of school.
- (4.15) **Underlying processes and mathematical tools.** The student communicates about Grade 4 mathematics using informal language.
- (4.16) **Underlying processes and mathematical tools.** The student uses logical reasoning.
- (B) use tools such as a clock with gears or a stopwatch to solve problems involving elapsed time.
- The student is expected to:
- (A) use concrete objects or pictures to make generalizations about determining all possible combinations of a given set of data or of objects in a problem situation; and
- (B) interpret bar graphs.
- The student is expected to:
- (A) identify the mathematics in everyday situations;
- (B) solve problems that incorporate understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
- (C) select or develop an appropriate problem-solving plan or strategy, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
- (D) use tools such as real objects, manipulatives, and technology to solve problems.
- The student is expected to:
- (A) explain and record observations using objects, words, pictures, numbers, and technology; and
- (B) relate informal language to mathematical language and symbols.
- The student is expected to:
- (A) make generalizations from patterns or sets of examples and nonexamples; and
- (B) justify why an answer is reasonable and explain the solution process.

### **§111.17. Mathematics, Grade 5.**

- (a) Introduction.
- (1) Within a well-balanced mathematics curriculum, the primary focal points at Grade 5 are comparing and contrasting lengths, areas, and volumes of two- or three-dimensional geometric figures; representing and interpreting data in graphs, charts, and tables; and applying whole number operations in a variety of contexts.

- (2) Throughout mathematics in Grades 3-5, students build a foundation of basic understandings in number, operation, and quantitative reasoning; patterns, relationships, and algebraic thinking; geometry and spatial reasoning; measurement; and probability and statistics. Students use algorithms for addition, subtraction, multiplication, and division as generalizations connected to concrete experiences; and they concretely develop basic concepts of fractions and decimals. Students use appropriate language and organizational structures such as tables and charts to represent and communicate relationships, make predictions, and solve problems. Students select and use formal language to describe their reasoning as they identify, compare, and classify two- or three-dimensional geometric figures; and they use numbers, standard units, and measurement tools to describe and compare objects, make estimates, and solve application problems. Students organize data, choose an appropriate method to display the data, and interpret the data to make decisions and predictions and solve problems.
- (3) Throughout mathematics in Grades 3-5, students develop numerical fluency with conceptual understanding and computational accuracy. Students in Grades 3-5 use knowledge of the base-ten place value system to compose and decompose numbers in order to solve problems requiring precision, estimation, and reasonableness. By the end of Grade 5, students know basic addition, subtraction, multiplication, and division facts and are using them to work flexibly, efficiently, and accurately with numbers during addition, subtraction, multiplication, and division computation.
- (4) Problem solving, language and communication, connections within and outside mathematics, and formal and informal reasoning underlie all content areas in mathematics. Throughout mathematics in Grades 3-5, students use these processes together with technology and other mathematical tools such as manipulative materials to develop conceptual understanding and solve meaningful problems as they do mathematics.

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(b) Knowledge and skills.

(5.1) **Number, operation, and quantitative reasoning.** The student uses place value to represent whole numbers and decimals.

The student is expected to:

- (A) use place value to read, write, compare, and order whole numbers through the 999,999,999,999; and
- (B) use place value to read, write, compare, and order decimals through the thousandths place.

(5.2) **Number, operation, and quantitative reasoning.** The student uses fractions in problem-solving situations.

The student is expected to:

- (A) generate a fraction equivalent to a given fraction such as  $\frac{1}{2}$  and  $\frac{3}{6}$  or  $\frac{4}{12}$  and  $\frac{1}{3}$ ;
- (B) generate a mixed number equivalent to a given improper fraction or generate an improper fraction equivalent to a given mixed number;
- (C) compare two fractional quantities in problem-solving situations using a variety of methods, including common denominators; and
- (D) use models to relate decimals to fractions that name tenths, hundredths, and thousandths.

- (5.3) **Number, operation, and quantitative reasoning.** The student adds, subtracts, multiplies, and divides to solve meaningful problems.
- The student is expected to:
- (A) use addition and subtraction to solve problems involving whole numbers and decimals;
  - (B) use multiplication to solve problems involving whole numbers (no more than three digits times two digits without technology);
  - (C) use division to solve problems involving whole numbers (no more than two-digit divisors and three-digit dividends without technology), including interpreting the remainder within a given context;
  - (D) identify common factors of a set of whole numbers; and
  - (E) model situations using addition and/or subtraction involving fractions with like denominators using concrete objects, pictures, words, and numbers.
- (5.4) **Number, operation, and quantitative reasoning.** The student estimates to determine reasonable results.
- The student is expected to use strategies, including rounding and compatible numbers to estimate solutions to addition, subtraction, multiplication, and division problems.
- (5.5) **Patterns, relationships, and algebraic thinking.** The student makes generalizations based on observed patterns and relationships.
- The student is expected to:
- (A) describe the relationship between sets of data in graphic organizers such as lists, tables, charts, and diagrams; and
  - (B) identify prime and composite numbers using concrete objects, pictorial models, and patterns in factor pairs.
- (5.6) **Patterns, relationships, and algebraic thinking.** The student describes relationships mathematically.
- The student is expected to select from and use diagrams and equations such as  $y = 5 + 3$  to represent meaningful problem situations.
- (5.7) **Geometry and spatial reasoning.** The student generates geometric definitions using critical attributes.
- The student is expected to identify essential attributes including parallel, perpendicular, and congruent parts of two- and three-dimensional geometric figures.
- (5.8) **Geometry and spatial reasoning.** The student models transformations.
- The student is expected to:
- (A) sketch the results of translations, rotations, and reflections on a Quadrant I coordinate grid; and

- (5.9) **Geometry and spatial reasoning.** The student recognizes the connection between ordered pairs of numbers and locations of points on a plane.
- (5.10) **Measurement.** The student applies measurement concepts involving length (including perimeter), area, capacity/volume, and weight/mass to solve problems.
- (5.11) **Measurement.** The student applies measurement concepts. The student measures time and temperature (in degrees Fahrenheit and Celsius).
- (5.12) **Probability and statistics.** The student describes and predicts the results of a probability experiment.
- (5.13) **Probability and statistics.** The student solves problems by collecting, organizing, displaying, and interpreting sets of data.
- (B) identify the transformation that generates one figure from the other when given two congruent figures on a Quadrant I coordinate grid.
- The student is expected to locate and name points on a coordinate grid using ordered pairs of whole numbers.
- The student is expected to:
- (A) perform simple conversions within the same measurement system (SI (metric) or customary);
- (B) connect models for perimeter, area, and volume with their respective formulas; and
- (C) select and use appropriate units and formulas to measure length, perimeter, area, and volume.
- The student is expected to:
- (A) solve problems involving changes in temperature; and
- (B) solve problems involving elapsed time.
- The student is expected to:
- (A) use fractions to describe the results of an experiment;
- (B) use experimental results to make predictions; and
- (C) list all possible outcomes of a probability experiment such as tossing a coin.
- The student is expected to:
- (A) use tables of related number pairs to make line graphs;
- (B) describe characteristics of data presented in tables and graphs including median, mode, and range; and
- (C) graph a given set of data using an appropriate graphical representation such as a picture or line graph.

(5.14) **Underlying processes and mathematical tools.** The student applies Grade 5 mathematics to solve problems connected to everyday experiences and activities in and outside of school.

The student is expected to:

- (A) identify the mathematics in everyday situations;
- (B) solve problems that incorporate understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness;
- (C) select or develop an appropriate problem-solving plan or strategy, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem; and
- (D) use tools such as real objects, manipulatives, and technology to solve problems.

(5.15) **Underlying processes and mathematical tools.** The student communicates about Grade 5 mathematics using informal language.

The student is expected to:

- (A) explain and record observations using objects, words, pictures, numbers, and technology; and
- (B) relate informal language to mathematical language and symbols.

(5.16) **Underlying processes and mathematical tools.** The student uses logical reasoning.

The student is expected to:

- (A) make generalizations from patterns or sets of examples and nonexamples; and
- (B) justify why an answer is reasonable and explain the solution process.