

SATEC

JUST
A
SWINGING
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Lesson created by Kenneth Mc Mullen

INTRODUCTION - ABOUT THE MATHEMATICS

A. STUDENT PERFORMANCE OBJECTIVES - TEKS CORRELATION

(b) Foundations of Functions

- (1) The student uses properties and attributes of functions and applies functions to problem situations. Following are performance descriptions.
 - (A) For a variety of situations, the student identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.
 - (B) In solving problems, the student collects data and records results, organizes the data, makes scatterplots, fits the curves to the appropriate parent function, interprets the results, and proceeds to model, predict, and make decisions and critical judgments.
- (2) The student understands the importance of the skills required to manipulate symbols in order to solve problems and uses the necessary algebraic skills required to simplify algebraic expressions and solve equations and inequalities in problem situations. Following are performance descriptions.
 - (A) The student uses tools including matrices, factoring, and properties of exponents to simplify expressions and transform and solve equations.
 - (B) The student uses complex numbers to describe the solutions of quadratic equations.
 - (C) The student connects the function notation of $y =$ and $f(x) =$.

(c) Algebra and Geometry

- (4) The student formulates equations and inequalities based on square root functions, and uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. Following are performance descriptions.
 - (A) The student uses the parent function to investigate, describe, and predict the effects of parameter changes on the graphs of square root functions and describes limitations on the domains and ranges.
 - (B) The student relates representations of square root functions, such as algebraic, tabular, graphical, and verbal descriptions.
 - (C) For given contexts, the student determines the reasonable domain and range values of square root functions, as well as interprets and determines the reasonableness of solutions to square root equations and inequalities.



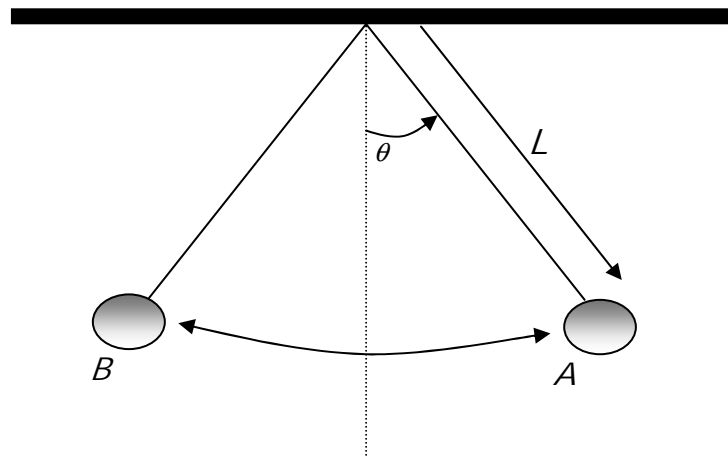
- (D) The student solves square root equations and inequalities using graphs, tables, and algebraic methods.
- (E) The student analyzes situations modeled by square root functions, formulates equations or inequalities, selects a method, and solves problems.
- (F) The student expresses inverses of quadratic functions using square root functions.

B. CRITICAL MATHEMATICS EXPLORED IN THIS ACTIVITY

A pendulum consists of a weighted object attached to a string or cord that is fastened to an apparatus that will allow the pendulum to freely swing or oscillate.

The physical properties or parameters of a pendulum that can affect its path and time of oscillation are the following:

- The length, l , of the pendulum (*In meters*)
- The mass of the object
- The angular displacement through which the pendulum swings
- The period, t , of the pendulum, which is the time it takes for the pendulum to swing through one complete oscillation (i.e. from point **a** to **b** and back to **a** in the figure below)



Also, it should be apparent that the period of oscillation of a pendulum is proportional to the square root of the length; for example, the longer the pendulum, the more time needed to complete one oscillation. Therefore,

the theoretical formula or expression for the period of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$ where g is the acceleration due to gravity or 9.80 m/s/s.

C. HOW STUDENTS WILL ENCOUNTER THE CONCEPTS

The student will be able to manipulate the parameters of an experimental set up and analyze the effects of these changes. Also, each student will become more familiar with the family of square root functions and their involvement in real world situations. The students will determine the experimental and theoretical values of the period of a simple pendulum through the manipulation of its length, and the involvement of technology, Graphical Analysis, Logger Pro 2.1, and the Photogate probe.

D. SETTING UP

The set up for the pendulum can be any stand that allows the objects to swing through the Photogate. Here, Stands from the Chemistry Dept. are used. With this kind of set up, the students can shorten the string by wrapping it around the clamp and then simply lowering the clamp along the rod. Washers taped together provide the weight at the end of the string.

MATERIALS NEEDED: Photogate probe, Graphical Analysis, Logger Pro, ruler, string, washers, stand and clamp or structure for the pendulum to oscillate



FRONT VIEW



SIDE VIEW

PACING: Time equivalency of one class period (approximately 1 hour 30 minutes) for instruction and completion of the experiment.

SUPPLEMENTARY COMMENTS:

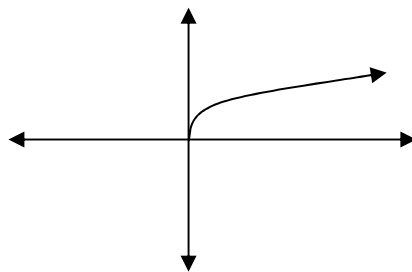
In this experiment, cooperative learning and teamwork are crucial to obtain accurate data and complete the assignment successfully.

F. ANSWERS (STUDENT ACTIVITY):

1. Length of the pendulum
2. Time to complete one oscillation

Length (cm)	Period (sec)
37	1.258
31	1.182
26	1.068
23	0.996
16	0.865

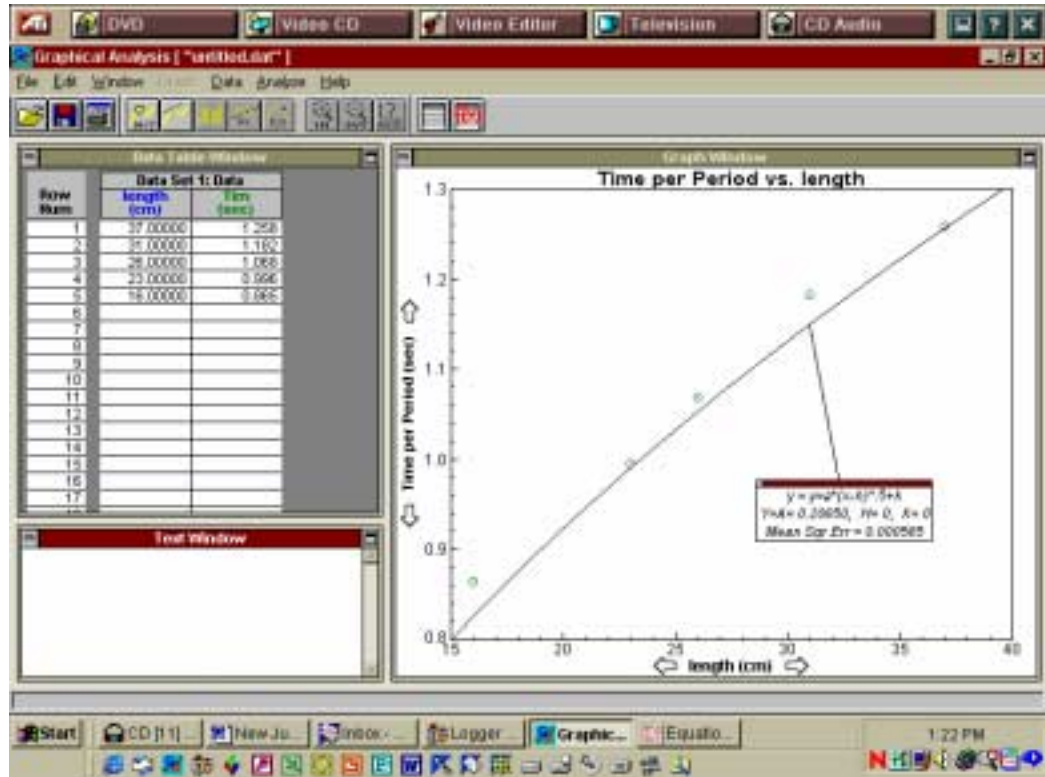
3. Explanation: Positive correlation, starting point at (0, 0), increases slowly over time, etc.



Answers may vary

4. Domain: $0 \leq x \leq 37$ Range: $0 \leq y \leq 1.258$
5. $y = .2065\sqrt{x}$ Mean Square Error: 0.000565

6.



7. $y = 2065\sqrt{22.4}$
 $y = 977$

$2 = 2065\sqrt{x}$

8. $9.685 = \sqrt{x}$
 $93.804 = x$

9. This is impossible because you cannot have a negative length.

10. 2 meters = 200 cm
 $y = 2065\sqrt{200}$
 $y = 2.920$

11. $0 = 2065\sqrt{x}$
 $0 = x$

12. Grandfather clock, Hypnosis, swinging boat ride at Six Flags, etc.

13. Answers may vary, but the experimental period should be very close to the theoretical period.

14. Answers may vary, but they should be less than 10 percent.

15. The answers should be pretty close together.

G. Extension

Find a grandfather clock at your home, relatives' home, or some other place of business, and measure or estimate the length of the pendulum and determine the time period. Repeat this process for a few more grandfather clocks and compare the time periods of each pendulum. Explain why the time periods differ for the various pendulums encountered and what factors cause these discrepancies.

F. Homework

Given the appropriate information and the needed equation, have the students complete a worksheet of 10 to 20 problems that are similar to the items found in **part II (5 through 9)** of the student activity sheet.

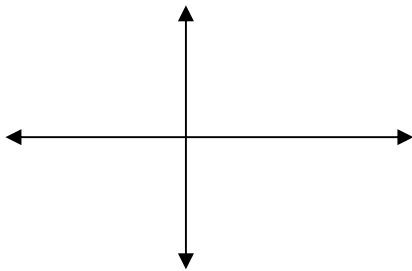


Introduction

In this experiment, you will investigate the relationship between the length of a pendulum and the time it takes for it to complete one oscillation (called its **period**).

For this experiment:

1. What is the **independent** variable? _____
2. What is the **dependent** variable? _____
3. Sketch a graph of what you think the relationship will look like.



Explain your reasoning. _____



PART I: Data Collection

Randomly select a long length for the pendulum. Measure the length of the string from the bottom on the clamp to the top of the washers in centimeters. Record the measurement in the table.

- Lay a ruler in front of the Photogate with the 10cm mark even with the pendulum. See Picture



- Open Logger Pro 2.1. Open the file called **Pendulum Timer**. (This is found under **Experiments-Probes & Sensors-Photogate**)
- Pull the pendulum to the left to the point that it is even with the end of the ruler.

- Click the  button and then release the pendulum. After it has taken about 10 readings click the  button. Record the reading for the 5th period in your table. (If you see inconsistent data within your readings, redo the experiment at the same length)
- To change the length of the pendulum, simply wrap the string around a part of the clamp. This should shorten the length.
- You will then have to adjust the clamp so that the washers hang directly above the bottom of the Photogate. The washers should not touch the Photogate. See Picture



- Measure the new length of the pendulum and record it in the table.
- Collect the new information for this length.
- Repeat the process until you have collected times for 5 different lengths.

Length (cm)	Period (sec)

4. State a reasonable Domain and the Range for the experiment. (Hint: What is the shortest possible length, longest possible length & shortest possible time, longest possible time)

Domain: _____

Range: _____

The parent function for this function is called the square root function and is written $y = a\sqrt{x-h} + k$. Now, using the **manual curve fit option**, type in the square root function. Reminder: $y = a\sqrt{x-h} + k$ is the same as $y = a(x-h)^{\frac{1}{2}} + k$. You will need to adjust the a, h, and k values until you can find the lowest mean square error as possible.

5. Write the equation you found. _____

STOP HERE! Check with your teacher before continuing. _____

6. Open **Graphical Analysis**, plot the data, turn off the connecting lines option, and **PRINT** a copy of the graph for each group member. Be sure to label your axes and units. The title of your graph should be your group member's names. Attach your graph to your paper to be turned in.

Use your equation to answer the following questions. Be sure to show your work!

7. What would be the time period for a pendulum with a length of 22.4 cm?
8. What would be the length of a pendulum that took 2 seconds to complete one oscillation?
9. What is the time period for a pendulum with a length of -9 cm. Explain your answer.
10. What is the time period for a pendulum with a length of 2 meters?
11. What would be the length of a pendulum that has a time period of 0 seconds? Explain your answer.
12. Where might you find the use of a pendulum in the real world?

By now you should have seen that the period of a pendulum depends on the length of the pendulum and on the acceleration of gravity at the pendulum's location. (That's 9.8 m/s/s on earth.) A man named Christian Huygens introduced a formula in 1673 that expresses the relation between the variables. This formula was useful in regulating the speed of clocks.

$T = 2\pi\sqrt{\frac{l}{g}}$, where T is the time for one oscillation, l is the length of the pendulum **in meters**, and g is the acceleration of gravity.

13. Use the formula above to calculate the theoretical period for 3 of your experimental lengths. (Don't forget to convert cm to meters)

Length:_____ Experimental period:_____ Theoretical period:_____

Length:_____ Experimental period:_____ Theoretical period:_____

Length:_____ Experimental period:_____ Theoretical period:_____

14. Find the Percent error for each of the three values you obtained above. To find the percent error you can use the following formula:

$Percent\ Error = \frac{|E - T|}{T} * 100\%$, where **E** represents the **Experimental Period**

and **T** represents the **Theoretical Period**.

1. _____
2. _____
3. _____

15. How close are the values? Is this reasonable? Explain your answer.
