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I About the Math

In this lesson the student will investigate the behavior of inverse functions by collecting data with the pressure sensor. Students will investigate concepts such as the domain and range of the parent function, parameter changes, continuity of the function, limits of the of the function, asymptotic behavior, as well as solving and writing rational equations.

II Setting Up

For this activity you will need **(1)** a copy of the file called “Stuff It!” (If you do not have a copy of the file, exp#30 from the **Physical Science with Computers** folder found in Logger Pro will work the same.), **(2)** a pressure sensor with the syringe attached to every computer, and **(3)** 1 balloon per group for Part I of the investigation. It is not necessary to have the cork attachment for this activity.

Attach a pressure sensor to the ULI in DIN 1. If you are using a LabPro, connect the pressure sensor to DIG 1.

When the students collect data for question # 5, the lever of the sensor has to be open, that is the lever will point directly towards the sensor. This will allow the students to fill the syringe with air. When the syringe is full, the students will have to close the valve, by turning the lever so that it points away from the sensor.

Be sure to tell students be consistent when reading the volume levels in the syringe, and to close the valve to maintain the pressure reading. This should provide better readings.

III Teacher Notes

a) Objective

The objectives of this lesson is to investigate the behavior of rational functions, to write equations given certain points, to study parameter changes, and to solve rational equations. Notions such as continuity, increasing/decreasing, and limits at the end points lend themselves to this lesson, and can be reviewed before, or can be discussed during the lesson.

b) Teachable Moments

- If you have not introduced concepts such as continuity of functions, increasing/decreasing functions, and “limit” as the x -value approaches a points of an interval, this experiment can give you the opportunity to discuss those topics with the students.
- Students may need review at the end of the exploration on writing and solving rational equations.
- After the students collect the data for two different starting volumes, talk about the different equations they are deriving, why the value for “ a ” is different and what makes that change in the equation. This can be an excellent opportunity to relate the initial value of the volume and the “ a ” value in the equations.
- This provides an excellent opportunity to teach the students how to write equations for rational equations and to find the constant of variation given the initial pressure reading and volume of air to be compressed. It is important that the teachers provides with this information before students proceed to the assessment questions.

c) Connections

Students may not be familiar with the notation for limits, although they might understand and recognize the behavior of the function as x gets very large in both directions, or as it gets very small. This can be a good time to talk about the notation

for limits and what a limit represents in order to familiarize students with the terminology as they prepare for precalculus.

d) Classroom Management Tips

Begin the activity with Part I as a whole group discussion before they go to the computer, and spend a little time reviewing the answers for this part before they proceed onto Part II.

Be sure to assign a different value to each group for the second data collection. You may want to show the students how to determine when the lever is open or closed and how they can tell that by looking at the readings on the screen, as well as noticing that the syringe will not try to “repel” the push.

e) Pre-requisites

Knowledge of Domain and Range, Dependent/Independent Variable, Knowledge of the Parent Function $y = \frac{a}{x}$, Solving Rational Equations (which can be reviewed concurrently with the lesson).

f) Questions

What causes the change in the “*a*” value for the equations when the starting volume in the syringe was 15 mL. versus the corresponding “*a*” value when the volume in the syringe is other than 15 mL? (i.e. 20 mL.)

Why does the pressure decrease when you pull the syringe to a value higher than the one you started with? (For instance, if your starting volume was 15 ml and you pull the syringe back to 20 ml.)

How are the graphs different for each of the two functions?

g) Supplementary Comments

- The value of the pressure “is” 1 at sea level. Therefore, the pressure sensor will bounce between .99 and 1 atm or 96 kPa.
1 atm = 101.325 kPa = 1013 millibar
- You might have to double check that they have the lever in the right position when taking measurements.
- *Logger Pro* does not deal as effectively calculating the pressure as the value in the volume approaches zero. Copying the data to *Graphical Analysis* might be considered since graphical analysis will actually give you a reading of “infinity” as the volume is near zero.

h) Follow-ups/extensions

A full investigation on Boyle’s Law can be given at the end of the activity. Students can be asked to research that law and consult with their Chemistry teacher.

i) Answers

PART I

- a) The volume increases, and it creates pressure in the inside of the balloon.
- b) The pressure increases inside the balloon as more air is blown into it.
- c) The pressure “pushing” inside the balloon is released returning the balloon to its normal state, and the pressure decreases back to the normal atmospheric pressure.
- d) The pressure should return to approximately 96.2 kPa. It will never be equal to zero.
- e) Because the pressure in the inside is higher than the pressure outside the balloon, therefore, the body is not able to retain the higher pressure causing it to lose it until the pressure outside that balloon is equal to the pressure inside the balloon.

- f) It “explodes” since the pressure built inside the can is higher than the pressure outside the can.
 g) It appears to be bigger than the size of the container.

Part II

Pressure	15	14	13	12	11	9	8	7	6	5
Volume	1.060	1.129	1.206	1.311	1.410	1.534	1.721	1.922	2.175	2.503

- 1) Volume (Independent Variable) and Pressure (Dependent Variable)
 2) Domain: $V > 0$ Range: $P > 0$

Pressure	19	18	17	16	15	14	13	12	11	10
Volume	1.035	1.085	1.143	1.201	1.272	1.344	1.435	1.539	1.661	1.798

- 3) The pressure will increase
 4) Pressure readings should be at 1 or near 1 atm or approx. 96.2 kPa. The pressure remains constant regardless of the size of the “container”.
 5) Approx 1.410
 6) No it cannot remain at 10 mL. because the pressure inside the syringe is trying to return to its natural state/value.
 7) The pressure built inside the syringe increases.
 8) The pressure inside the syringe decreases since the “container” size is increased, and causing the pressure to drop below the pressure outside the syringe

Sample Data with 15 mL of air:

Sample Data with 20 mL of air:

PART III

- 9) Rational Function; $y = \frac{a}{x}$
 10) **Domain:** $-\infty < x < \infty ; x \neq 0$ **Range:** $-\infty < y < \infty ; y \neq 0$
 11) For this situation there is a restricted domain which is all positive real numbers greater than zero.
 12) **Equation for 15 mL:** $y = \frac{13.431}{x}$ or $y = \frac{1360.896}{x}$ (depending on the units)
Equation for 20 mL: $y = \frac{18.689}{x}$ or $y = \frac{1893.663}{x}$
 13) Yes, they are both rational functions. The “a” value is different for both equations. The difference in the “a” parameter is caused by the initial amount in the syringe which causes the “a” value to either increase or decrease.
 14) Yes, the vertical asymptote is the y-axis, and the horizontal asymptote is the x-axis.
 15) $f(x) \rightarrow \infty$
 16) $f(x) \rightarrow 0$
 17) (answers will vary)

	First Equation	Second Equation
2.5 mL	5.372	7.476
4 mL	3.358	4.672
30 mL	0.448	0.623

18) (answers will vary) The second equation here is at 20 mL

	First Equation	Second Equation
0.5 A	26.862 mL	37 mL
250 A	0.054 mL	0.075 mL
650 A	0.021 mL	0.029 mL

PART IV

19) $y = \frac{23.725}{x}$ or $y = \frac{2403.936}{x}$

20) When $V = 12$, then $P = 2.083$; When $V = 27$, then $P = 0.926$

21) When $P = 900$, then $V = 0.028$; When $P = 0.5$, then $V = 50$

22) It is an Indirect relationship.

V **TEKS**

(c) Knowledge and skills.

(1) The student defines functions, describes characteristics of functions, and translates among verbal, numerical, graphical, and symbolic representations of functions, including polynomial, rational, radical, exponential, logarithmic, trigonometric, and piecewise-defined functions. The student is expected to:

(A) describe parent functions symbolically and graphically, including $y = x^n$,

$$y = \ln x, y = \log_a x, y = \frac{1}{x}, y = e^x, y = a^x, y = \sin x, \text{ etc.};$$

(B) determine the domain and range of functions using graphs, tables, and symbols;

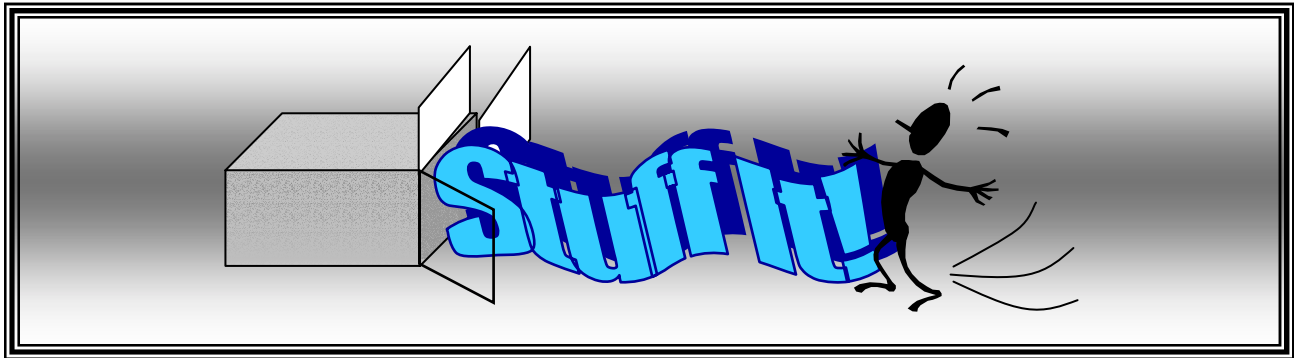
(E) investigate continuity, end behavior, vertical and horizontal asymptotes, and limits and connect these characteristics to the graph of a function.

(3) The student uses functions and their properties to model and solve real-life problems. The student is expected to:

(A) use functions such as logarithmic, exponential, trigonometric, polynomial, etc. to model real-life data;

(B) use regression to determine a function to model real-life data;

(C) use properties of functions to analyze and solve problems and make predictions;



Many times, in real life we experience situations in which a certain amount of mass, whether is it is a solid or a gas, is to be stored in a container which is smaller in volume. Such is the case of air stored in a cylinder. A gas is compressed in order to build pressure which can be used to operate machinery such as a pump for a swimming pool, steam for a train, etc. At times we are interested in knowing the amount of air necessary to create a certain amount of pressure to operate such machinery, or what the pressure will be after the volume is compressed into half its original volume.

PART I. Introduction

- a) What happens to the volume of a balloon when air is blown into it without releasing the air?

- b) How does the pressure inside the balloon change as you blow more and more air into the balloon?

- c) What happens to the pressure inside the balloon as you let go of the end of the balloon so as to let air come out?

- d) What should be the pressure inside the balloon as all the air is released?

- e) Why does the balloon loose the air when you release your hold from the balloon spout?

- f) What happens when you open a can of biscuit-dough? What causes that reaction?

- g) How does the amount of dough compare to the size of the package?

PART II. Collecting the data

During this experiment, you will investigate the relationship between the pressure created as an amount of air is “compressed” into a smaller volume.

- **Launch** Logger Pro and **Open** the file called Stuff It!
- Verify that the valve is open. (Make sure the handle label “off” points to the probe’s box.)

1. Identify the two variables for this experiment, and indicate which variable is the **Independent variable**, and which is the **Dependent variable**, and explain why.

Independent:

Dependent:

2. Identify an appropriate **Domain** and **Range** for this situation.

Domain:

Range:

3. From your previous knowledge, how does the pressure change as you “compress” the air into a smaller volume?

4. **Without closing the valve, change the amount of air in the syringe three times and obtain the pressure reading for the respective volume.**

(i) Volume: ___ Pressure ___ (ii) Volume: ___ Pressure ___ (iii) Volume: ___ Pressure: _____

What can you conclude about the atmosphere’s pressure around you?

Begin the experiment by allowing the syringe to fill with air to a volume of 15 mL, and then closing the valve.

5. Compress the air volume to 10 mL. What is the pressure reading now?
6. Determine what happens if you release the syringe after compressing the air to 10 mL? Explain.
7. Explain what happens to the pressure as you push the syringe to 10 mL?
8. Explain what happens to the pressure as you pull the syringe to 20mL? Explain why.

Beginning with a volume of 15 mL, collect 10 samples of pressure for different volumes, and record the data collected in the table provided. Be sure to round to 3 decimals.

Data with 15 mL of air:

Pressure										
Volume										

Repeat the same procedure as above, but for your group, use the volume of air assigned by the teacher. Fill the table in the space provided.

Data with ___ mL of air:

Pressure										
Volume										

PART III. Applying the Mathematics

9. Apply your previous knowledge to identify the type of function for this relationship, and give the parent form for that function type.

10. What is the **Domain** and the **Range** for that **function**?

Domain:

Range:

11. What values of the domain for the parent function are meaningless for our pressure/volume situation ?

12. **Using Logger Pro**, analyze the data you collected and perform a curve fit using the function that best represents both sets of data. Write your equations in the space below.

Equation for 15 mL:

Equation for ___ mL:

13. Are there any similarities in the equations? How are they different? Apply what you know about parameter changes to explain what causes the difference(s) in the equations.

14. Are there any asymptotes for this situation? If so, where are they located?

15. Based on your data collected, what happens to the y-value of the function as the value of x approaches zero?

Using **limit** notation, we can write: $\lim_{x \rightarrow 0} f(x) = \underline{\hspace{2cm}}$

16. Based on your data collected, what happens to the y-value of the function as the value of x approaches infinity?

Using **limit** notation, we can write: $\lim_{x \rightarrow \infty} f(x) = \underline{\hspace{2cm}}$

17. Use **both equations you obtained during your data collection** and find the pressure when the air is “compressed” into a 2.5 mL, 4 mL, and 30 mL volume.

First Equation

Second Equation

18. Using **both equations you obtained during your data collection**, find the respective volume when the pressure is 650 A, 250 A, and 0.5 A.

First Equation

Second Equation

PART IV. Assessment. Turn in your work on a separate piece of paper. Be sure to include all your work

19. Based on your investigations, write an equation that would be represent the relationship between pressure and volume if you start with a volume of 25 mL of air. Explain how you arrived to this equation.
20. Using the equation above, predict the pressure when the volume is compressed to 12 mL.
27ml.
21. Predict the Volume when the pressure reading is 900 atm, 0.5 atm.
22. Based on your findings, how is the pressure related to the volume of air? Is this a direct or indirect relationship? Why?