

# San Antonio Technology In Education Coalition

Over Da Watta!



**Building a Bridge Using Parabolas**

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*Lesson created by Andy Sorensen*

## A. Students Performance Objectives

### TEKS

(d) Quadratic and square root functions: knowledge and skills and performance descriptions.

- (1) The student understands that quadratic functions can be represented in different ways and translates among their various representations.
  - (B) The student relates representations of quadratic functions, such as algebraic, tabular, graphical, and verbal descriptions.
  - (C) The student determines a quadratic function from its roots or a graph.
- (2) The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations.
  - (A) The student uses characteristics of the quadratic parent function to sketch the related graphs and connects between the  $y = ax^2 + bx + c$  and the  $y = a(x - h)^2 + k$  symbolic representations of quadratic functions.
- (3) The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.
  - (A) The student analyzes situations involving quadratic functions and formulates quadratic equations or inequalities to solve problems.
  - (B) The student compares and translates between algebraic and graphical solutions of quadratic equations.
  - (C) The student solves quadratic equations and inequalities.

## B. Critical Math Explored

- The principal topic of this activity is exploring quadratic functions in standard (or vertex) form.
- Students will use Scion Image to collect data from a picture of the Hakata Bridge. They will then use the graphing calculator to make a scatterplot and write the equation of the suspension cable of the bridge.
- Students will learn the geometric names of the parts of quadratic functions, notion of minimum and maximum, concavity, and roots.

- In the Reflect and apply, the students will be asked to find the roots for a third parabola.
- Students will reinforce their knowledge of linear functions, slopes (vertical, horizontal lines), and will write equations for vertical and horizontal lines.

### C. Teacher Notes

#### a) Objective

This is an investigation of how the arch of the Hakata Bridge creates a parabola. As a class, demonstrate that there are 6 equations needed to form this bridge.

(1) the suspension arch (2) the driving lanes, (3 and 4) the two vertical towers, and (5-6) the two diagonal support cables.

Relate how the intersections of the driving lane and the arch can be considered the root of the quadratic function. Also relate how the intersection of the vertical supports and the arch are the intersection of two systems of equations.

#### b) Teachable Moments

A discussion for the meaning of roots, the number of roots and in general a discussion of the discriminant is recommendable.

The notion of Concavity and Minimum and Maximum will also be analyzed. Students will need reinforcement on these ideas as they work through the lesson and after completing the lesson.

Ideas on how parameter changes will affect the shape and location of the graph are also discussed at the end of the lesson.

### D. Connections

Another objective of this lesson is to make connection between the number of roots of an equation and the equation of the arch, the meaning of the root(s) in problem solving, and the terminology.

Be sure to point out to students that changing the  $a$  value will create a different function that will have the same root(s). Therefore the importance of understanding that there are infinitely many functions that have the same roots.

The White boxes contain topics that most likely are new to the students. It will be important that the teacher emphasizes that information.

#### c) Classroom Management Tips

It may be helpful to work through this lesson closely as a group, that is, assign 1-3 in the first 20 minutes and review together; next assign 4-5 in the next 10 minutes and review; and so on.

#### d) Pre-requisites

- The student should be familiar with the equations of vertical and horizontal lines.
- The student should be familiar with the statistical capability of the graphing calculator.
- The student should be familiar with parameter changes.
- The student should have previous knowledge of what a quadratic function looks like, and the parent function.

#### e) Questions

How many root(s) are in this bridge and what are they?

What does the root represent in terms of the parabola?

What else does the vertex represent in this case?

If the arch was “flipped upside down, how many roots would there be?--

How would that affect the equation of the arch?

How many equations can have the same roots?—What makes them different from each other?

How does the “a” value of the equation affect the shape of the graph?

How does changing the length of the main span affect the equation?

#### E. Setting Up

Every computer will need to have Scion Image, the Over Da Watta! File, and the corresponding macro. Graphing calculators will also be needed. Be sure the files are in every computer.

#### F. **What the Teacher Should do to Prepare**

Be sure to review the student activity previous to class time. The lesson was formatted and some questions have changed or altered. Look at the supplementary comments for helpful hints.

Prepare a similar problem like question 16 to work with the class before assigning the Reflect and Apply Assignment.

#### G. **The Teacher’s Perspective**

The lesson presents a lot of new information to the students. It will be very critical that the teacher reinforces these topics throughout the study of the quadratic unit.

#### H. **Supplementary Comments**

- Remind the students that the roadway is  $y = 0$ .
- Under Options, be sure to invert the y-coordinate from Preference.
- Be sure they select both Perimeter/length and X-Y Coordinate from Analyze in the menu bar.
- In order to collect data, the students will have to set the scale measuring the known distance for the main span of the bridge, which is 560 meters.

- Their first measurement should be the new origing with respect to the picture.
- The next point should be the top of the left vertical support followed by random points that is part of the suspension cable.
- You may need to guide them in finding the equation of the line representing the driving lanes using the point of intersection of the arch and the driving lanes. Since the height is zero and the line is horizontal, the ordered pair is  $(x, 0)$ , and the equation of the line is  $y = 0$ .
- On their table, the only two values that they should have are the X-Coordinate and the Height of the vertical support cable above the driving lane. The **X-Coordinate tells you how far that point is from the left end of the bridge.**
- It is very important that you stress to them to be consistent about clicking on the actual arch in order to more accurately determine the height of the point on the suspension arch.

## I. Answers and Notes.

- 1) 6 parts of the bridge. The horizontal line is the Driving Lane, The Vertical Lines (2) are the Vertical Supports, The Diagonal Lines (2) are the Diagonal Supports, and The Arch is the Suspension Bridge.
- 2) Quadratic Function. 3 Intersections.
- 3) Zero since this is considered ground level.
- 4) Slope is zero since it is a horizontal line.
  - (b) The equation is  $y = 0$
- 5) (a) Concave Up since it opens up.
  - (b) Since the arch is concave up it will have a minimum.
  - (c) This is also the vertex of the parabola.
- 6) Answers will vary. X values should range between 150 – 730, and the y-values should range between 0 and 70. Any other values will indicate that they probably did not set the scale properly.
- 7) Only one since it is assumed that the bridge is only connected once to the driving lane.
  - (a) Approx. 150 meters.
  - (b) Approx. 60 meters.
  - (c)  $(150, 60)$
  - (d) It should be  $x = 150$  based on this data. Look at the data collected to determine the equation.
  - (e)  $(710, 60)$
  - (f)  $x = 710$
- 8) Because they are vertical lines, their slopes should be zero. Their equations become  $x = a$ ; where  $a =$  the x-intercept.
- 9) Right to Left:  $(0, 0)$  ,  $(150, 60)$  ,  $(430, 0)$  ,  $(710, 60)$

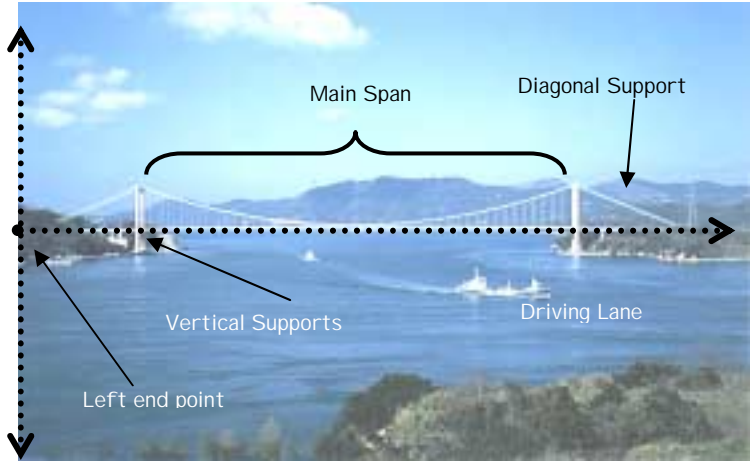
- 10) There is only one root since it is assumed that the arch touches the driving lane only at one point.
- 11) The vertex coordinate (Answers will vary but should be close to 430)
- 12) Root = X-intercept = Zero = Solution.
- 13) Yes. The Only thing that needs to change is the " $a$ " value as long as the roots are kept constant.
- 14) Answers will vary. Check for the same root value.
- 15)  $y = 0.000765(x - 430)^2$ . Answer will vary accordingly to their root.
- 16) (a) If the Vertical supports are only 35 meters then that would cause for the graph to open up wider.  
 (b) Since the y values only increase to 35 instead of 60. this will cause for the  $a$  parameter to change.
- 17) If the main span changes to 800 meters, this would cause also to open wider, hence changing the  $a$  value. Most importantly is the change in the location of the vertex which will cause for the graph to shift to the right.
- 18) Answers will vary. It is recommended you create your own rubric that reflects the information that you emphasized during the activity.
- 19)  $y = 0.00064(x - 325)^2$   
 (a) The slope is undefined since it is a vertical line.  
 (b) The two equations are  $x = 132$  and  $x = 712$ .

## J. Extensions

There is a Reflect and Apply that reinforces the concepts learned in the lesson. It would be very helpful to do an exercise in class similar to exercise in question 16 and pose the question of how the problem would change If the arch was concave down before you assign the reflect and apply.

# Over Da watta!

Bridges come in many forms and can be constructed in many styles. One of the very attractive and common designs in engineering is the suspension bridge. One of the reasons that this type of bridge is a favorite among engineers is its strength and design. An example of this type of bridge is the world



famous Golden Gate Bridge in San Francisco. In this investigation, you will analyze a bridge located in Japan called the Hakata Bridge. Like the Golden Gate Bridge, The Hakata Bridge is a suspension bridge.

## PART I. Collecting the information

1. Look at the picture. How many lines and/or curves create the structure of the Hakata Bridge? *Identify each as a part of the bridge. (e.g. the horizontal line is the driving lanes).* \_\_\_\_\_

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2. Analyze the suspension arch of the bridge. What parent function is represented by this arch? \_\_\_\_\_ Circle the points where the arch intersects any part of the bridge in the picture. How many intersections are? \_\_\_\_\_

For this situation, we will consider *the driving lanes to be ground level*. Therefore, anything below the bridge will be considered underground (and therefore negative). **Assume the suspension arch is supposed to touch the driving lane at one point only.**

3. (a) What is the height at that point in relation to the driving lanes? \_\_\_\_\_

4. (a) What is the slope of the line representing the *driving lane*? Explain how you arrived to this conclusion or show your work. \_\_\_\_\_

(b) What is the equation of the line? \_\_\_\_\_

When working with curves such as parabolas, it is sometimes important to know what the lowest point (the minimum), or highest point (the maximum), of a curve will be. In this situation the parabola has a minimum point.

A curve is said to be **Concave Up** if the graph has a "valley" or **Concave Down** if it has a "hill". Therefore, we can conclude the following:

(a) If a parabola is *concave up* it has a *minimum*.

(b) If a parabola is *concave down* it has a *maximum*.

5. (a) What is the concavity of the parabola representing the arch of the bridge? \_\_\_\_\_

(b) Is this representing a maximum or a minimum? \_\_\_\_\_

(c) What is another name for the minimum/maximum of a parabola? \_\_\_\_\_

6. Using Scion Image, collect twenty points on the curve, that is the suspension arch. **Record your data in the table below.**

Distance from the left end of the bridge																			
Height of the suspension cable above the roadway																			

7. Answer the following questions based on the data you have collected and what you know to this point about the bridge.

(a) Determine the distance from the left end point to the left vertical support.

(b) Determine the height of the left vertical support from "ground level". \_\_\_\_\_

(c) What is the coordinate of the top of the left vertical support? (      ,      )

(d) What is the equation representing the left vertical support? \_\_\_\_\_

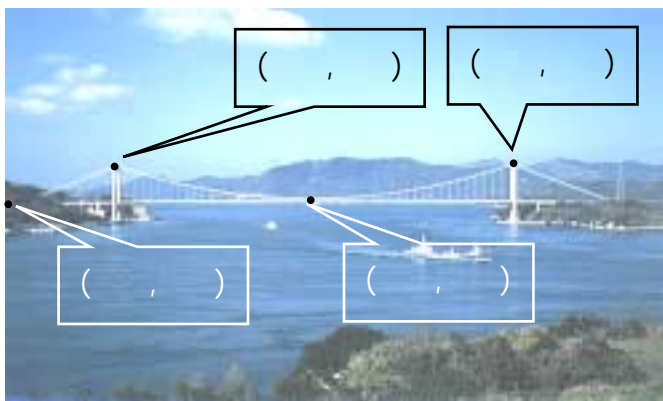
(e) What is the coordinate of the top of the right vertical support?(      ,      )

(f) What is the equation representing the right vertical support? \_\_\_\_\_

8. Explain how you were able to determine the equations for the vertical supports?

What do you know about their slopes? Intercepts?

9. Label the corresponding coordinate points.



**Part II. Finding the equation of the suspension arch of the bridge in vertex (or standard) form.**

***Roots*** of a curve can help us determine the equation of a curve, in this case a parabola. Another name for the Root is "*the zero of a function*" or "*the solution of an equation*". ***This/these point(s) occur when the value of the function ("y") is zero.***

10. How many "Root(s)" exist for the parabola in the bridge? Justify your answer.

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11. What is/are the root(s) of this parabola? \_\_\_\_\_

12. What are the three synonyms for the "roots" of a function? \_\_\_\_\_

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Once we know the roots of a curve, we can find *an equation* whose roots are these points. The general form would be  $y = a(x - r_1)(x - r_2)$ ; where  $r_1$  and  $r_2$  represent the roots of the quadratic.

In some quadratics there is only one unique root (but there are still 2 roots). In this case  $y = a(x - r_1)(x - r_1)$  which is the same as  $y = a(x - r)^2$ .

13. Is it possible to have more than one equation with the same root(s)? Explain.  
(HINT: What part of the equation would remain the same? What would change?)
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14. Create the equations of three parabolas that have the same root as the quadratic equation of the suspension arch of the Hakata Bridge.

a)

b)

c)

**Find an equation for the parabola representing the suspension arch.**

- i) Use your graphing calculator to draw a scatter plot of the data you collected in **question 6**.
- ii) Graph one of the equations you obtained in question 14. You should be able to see both the scatterplot and the equation.
- iii) Apply your previous knowledge about parameter changes to make the graph fit the scatterplot as closely as possible.
- iv) Write the equation that best fits the parabola for the suspension arch.

15. **Bridge Equation** : \_\_\_\_\_

16. (a) Assume that the height of the tower is only 35 m. How would the **graph** of the parabola be affected? \_\_\_\_\_

(b) What parameter would be responsible for that change? Explain why. \_\_\_\_\_

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19. **Draw a diagram or sketch** of the suspension arch for a bridge similar to the Hakata Bridge given the conditions below. **Label the dimensions and intersections. Find the equation, in vertex form,** of the suspension arch.

- (a) The distance between the two vertical towers is 500 meters.
- (b) The highest point of the tower is 40 meters.
- (c) The driving lanes extend 75 meters from each side of the towers.