San Antonio Technology In Education Coalition

Mathematics in the Shadows

This lesson was developed under a grant funded by the United States Department of Education, Office of Education Research and Improvement.

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Mathematics in the Shadows

A. Correlation to State Assessment (TEKS):

(b) Foundations for functions:

(1) The student uses properties and attributes of functions and applies functions to problem situations.

(A) For a variety of situations, the student identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

(B) In solving problems, the student collects data and records results, organizes the data, makes scatterplots, fits the curves to the appropriate parent function, interprets the results, and proceeds to model, predict, and make decisions and critical judgments.

(c) Algebra and geometry:

(1) The student connects algebraic and geometric representations of functions.

(A) The student identifies and sketches graphs of parent functions, including linear ($y = x$), quadratic ($y = x^2$), square root ($y = \sqrt{x}$), inverse ($y = 1/x$), exponential ($y = ax$), and logarithmic ($y = \log_a x$) functions.

(B) The student extends parent functions with parameters such as m in $y = mx$ and describes parameter changes on the graph of parent functions.

(d) Quadratic and square root functions:

(1) The student understands that quadratic functions can be represented in different ways and translates among their various representations.

(A) For given contexts, the student determines the reasonable domain and range values of quadratic functions, as well as interprets and determines the reasonableness of solutions to quadratic equations and inequalities.

(B) The student relates representations of quadratic functions, such as algebraic, tabular, graphical, and verbal descriptions.

(C) The student determines a quadratic function from its roots or a graph.

(2) The student interprets and describes the effects of changes in the parameters of quadratic functions in applied and mathematical situations.

(A) The student uses characteristics of the quadratic parent function to sketch the related graphs and connects between the $y = ax2 + bx + c$ and the $y = a(x - h)2 + k$ symbolic representations of quadratic functions.

(B) The student uses the parent function to investigate, describe, and predict the effects of changes in a, h, and k on the graphs of $y = a(x - h)2 +k$ form of a function in applied and purely mathematical situations.

(3) The student formulates equations and inequalities based on quadratic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

(A) The student analyzes situations involving quadratic functions and formulates quadratic equations or inequalities to solve problems.
(D) The student solves quadratic equations and inequalities.

B. About the Mathematics

Critical Mathematics explored in this activity:

In this activity, the student will explore the length of a shadow over time. The student will investigate the effect of changing the angle at which the light source is directed and how it changes the length of a shadow over time. The student will identify independent and dependent variables, identify reasonable domain and range, and use their knowledge of parameter changes to fit a curve to quadratic data. The student will make predictions from the identified

models as well as make predictions of how changing a variable affects the model.

C. How students will encounter the concepts:

Students will examine the lengths of shadows over a time period using Scion Image (or NIH Imaging). After collecting data, students will use graphical analysis or a graphing calculator, and their knowledge of parameter changes to fit a curve to the data. The curve will take the form $y = a (x - h)^2 + k$.. Students will use the equation to make predictions and inferences.

B. Set Up

Materials:

- Lightware Projector connected to computer for demonstration
- Three shadow tif stacks
- Graphical Analysis or graphing calculator
- Scion Image or NIH Imaging
- Student Activity Sheets

C. Teacher Notes

Objectives:

- The student understands that quadratic functions can be represented in different ways
- The student determines reasonable domain and range
- The student relates representations of quadratic functions, such as algebraic, tabular, graphical, and verbal descriptions
- The student determines a quadratic function from its roots or a graph
- The student analyzes situations involving quadratic functions and formulates quadratic equations to solve problems
- The student compares and translates between algebraic and graphical solutions of quadratic functions

Teachable moments:

Students may need help determining the minimum and maximum of a quadratic.

Connections:

Be sure to make the connections in the use of roots from Over Da Watta to this lesson.

This lesson reinforces the general concept of parameter changes to the quadratic functions.

Classroom management tips:

- Part 1, watching the video clips and answering questions, can be done as a class discussion before the students break into groups to collect their own data.
- It is recommended that you have a flashlight and an object to project a shadow as a model to illustrate the situations represented in each video file.

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Prerequisite knowledge/skills:

- Understanding of the quadratic function from the survey
- Use of Scion Image (or NIH Imaging), Graphical Analysis (or graphing calculator)
- Terms: roots, vertex, parameter changes, domain, range, independent and dependent variables,

Performance based questions for assessment of student understanding:

Emphasize questions 13, 22, and 28-32.

H. Supplementary Teacher Comments:

- *When showing the three video files, the students need to be informed that the angle of the light source is different in all three video files.*
- In this experiment, the students will look at the quadratic relationship between time and the length of a shadow that is formed by a moving light source. The three files that the students will be examining were created by filming the shadow of a peg as a light source from above is moved across an arch. (see figure 1).

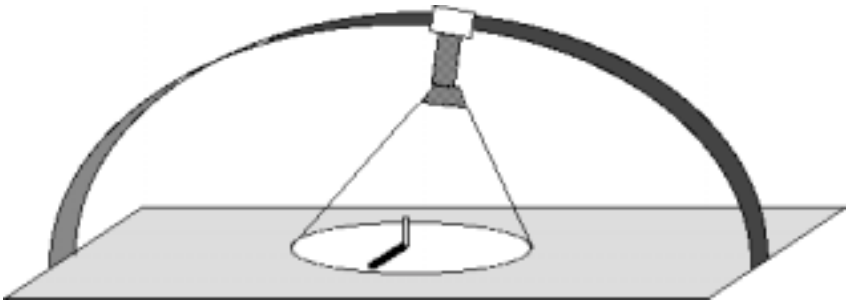
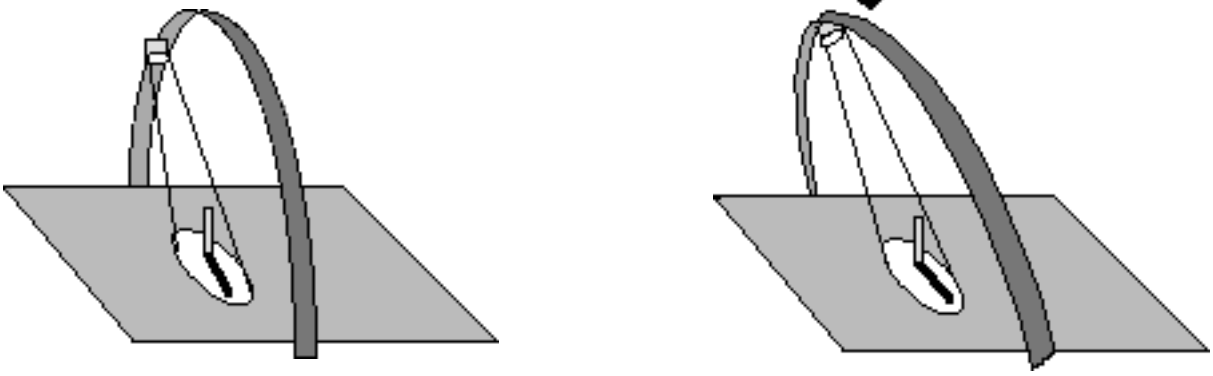


Figure 1

- For the first shadow, the arch was at a 90°angle to the base. For the second and third shadow, the arch was tilted at approximately 80° and 70° respectively.



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Part I

- It is suggested that the teacher show the movie of the first shadow to the entire class. Then each student would answer questions 1-5 individually. Next, the teacher would show the movie of the second shadow, and the students would individually answer questions 6-8.

Part II

- The contrast of the images of the movie can be adjusted by using the lute tool,  .
- The students will be gathering the data from the three movies. The independent variable will be time. Since the movie is recorded at 10 frames per second, every ten frames will be one second. It will be important that you check the students' work so that this measurement is accurate. The dependent variable will be the length of the shadow, measured in pixels.
- The curve fitting in Graphical Analysis will give the students a chance to use their knowledge of parameter changes on quadratic functions. The questions that follow are meant to cause the students to think about their choices for **a, h,** and **k,** as well as lead them into an understanding of their practical application to the real-world situation.
- The teacher should be familiar with the questions and responses so that spot checks can be made periodically to ensure the students are on the right track.

I. <u>Answers:</u>

1. Independent: Time in seconds
   Dependent: Length of shadow in pixels
2. The length of the shadow decreases for a time and then begins to increase.
3. See figure 1.
4. Because the length of the shadow decreases and then

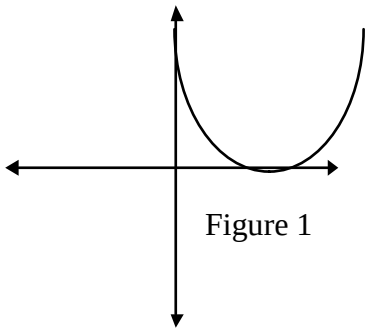

Figure 1

increases.

5. Yes there is one root for this function and it will happen when the shadow has zero length, or at the vertex.
6. See figure 2.

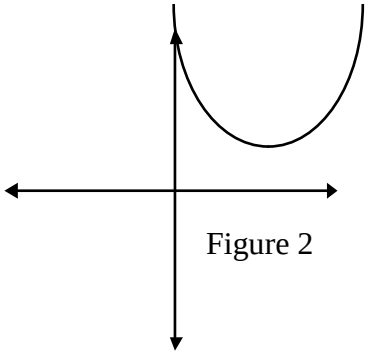

Figure 2

7. The second graph is shifted up a bit because the shadow is longer at the vertex of the second shadow than it is on the first shadow.

8. The second graph is moved higher then the first.

First Shadow ***Second Shadow*** ***Third Shadow***

Time	Length
sec	Pixels
0.00	44.6
1.00	26.6
2.00	14.6
3.00	7.58
4.00	3.58
5.00	3.42
6.00	9.42
7.00	17.5

Time	Length
sec	Pixels
0.00	31.8
1.00	25.6
2.00	22.0
3.00	16.1
4.00	14.8
5.00	12.3
6.00	14.1
7.00	18.5
8.00	22.1

Time	Length
sec	Pixels
0.00	79.4
1.00	63.4
2.00	50.5
3.00	43.7
4.00	37.5
5.00	35.8
6.00	37.3
7.00	38.7
8.00	44.4

9. Answers may vary depending on data…
10. (a) $y = 2.11(x-4.47)^2+3$
    (b) .367
11. "a" has to be a positive number since the parabola opened up. The larger the value for
    "a" the thinner the parabola was, the smaller the value for "a" the wider the parabola was.
12. These represent the vertex of the parabola.
13. The time when the shadow was the shortest.
14. 10.65 pixels
15. 0.220 seconds and 8.59 seconds

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16. (a) $y = .862(x-4.72)^2+13.6$
    (b) 0.958
17. (a) (4.72, 13.6)
    (b) The graph has been shifted up, and therefore there is a drastic change in the "y" value of the vertex.
18. $y = a(x-h)^2 + k$
    $y = a(x-4.72)^2 + 13.6$
    $use(5.2, 13.8)$
    $13.8 = a(5.2-4.72)^2 + 13.6$
    $13.8 = a(0.48)^2 + 13.6$
    $13.8 = a\,0.2304 + 13.6$
    $13.8 = a\,0.2304 + 13.6$
    $0.2 = 0.2304a$
    $0.868 = a$

19. Since the vertex is the same, the only comparison is the "a" value, which is just about equal in value.
20. 17.85 pixels
21. –0.803 seconds and 10.2 seconds
22. (a) The second graph is shifted up from the x-axis.
    (b) Because the shadow was longer in the second graph at its shortest point.
23. (a) $y = 1.49(x-5.39)^2+35$
    (b) 0.793
24. (a) (5.39, 35)
    (b) The graph was shifted up higher yet.
25. Since the "a" values range from .8 to 2.1 we can assume they are similar and that is because the graph is simply move up, but its shape has not changed. It is an indication that the light source was consistent.
26. 3.57 seconds and 7.23 seconds
27. 47.5 pixels
28. The third graph is shifted higher than the second graph.
29. The light source was more at an angle on the third graph than it was on the second graph.
30. The graph would be shifted up even further.
31. The shadow would always be infinitely long and therefore there would be no graph.
32. Just the first function had a root. The second and third do not because the shadow's length is always positive.

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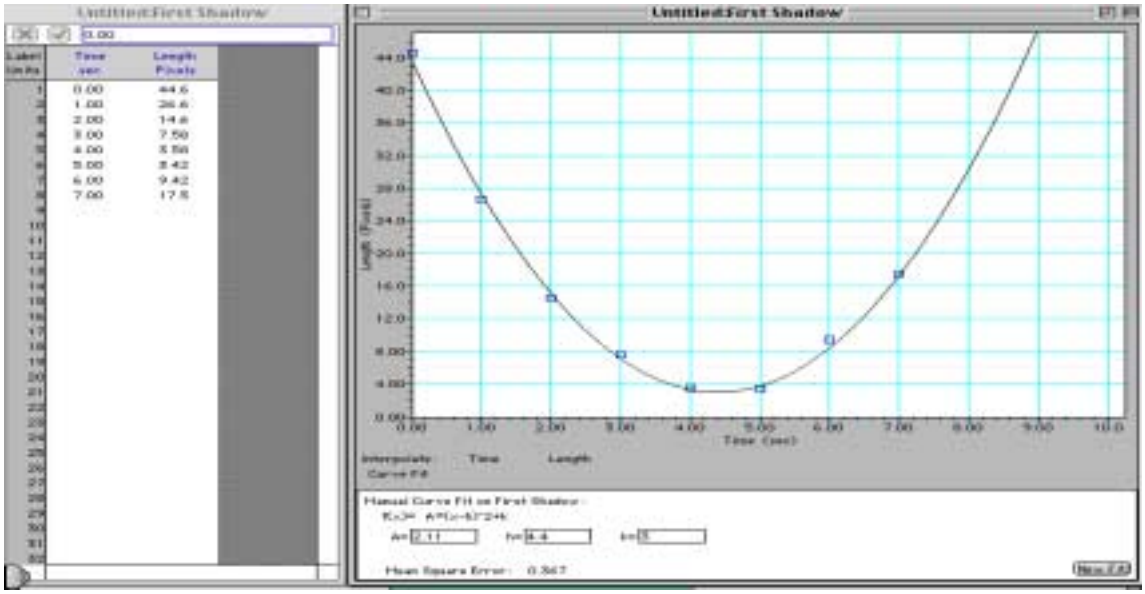

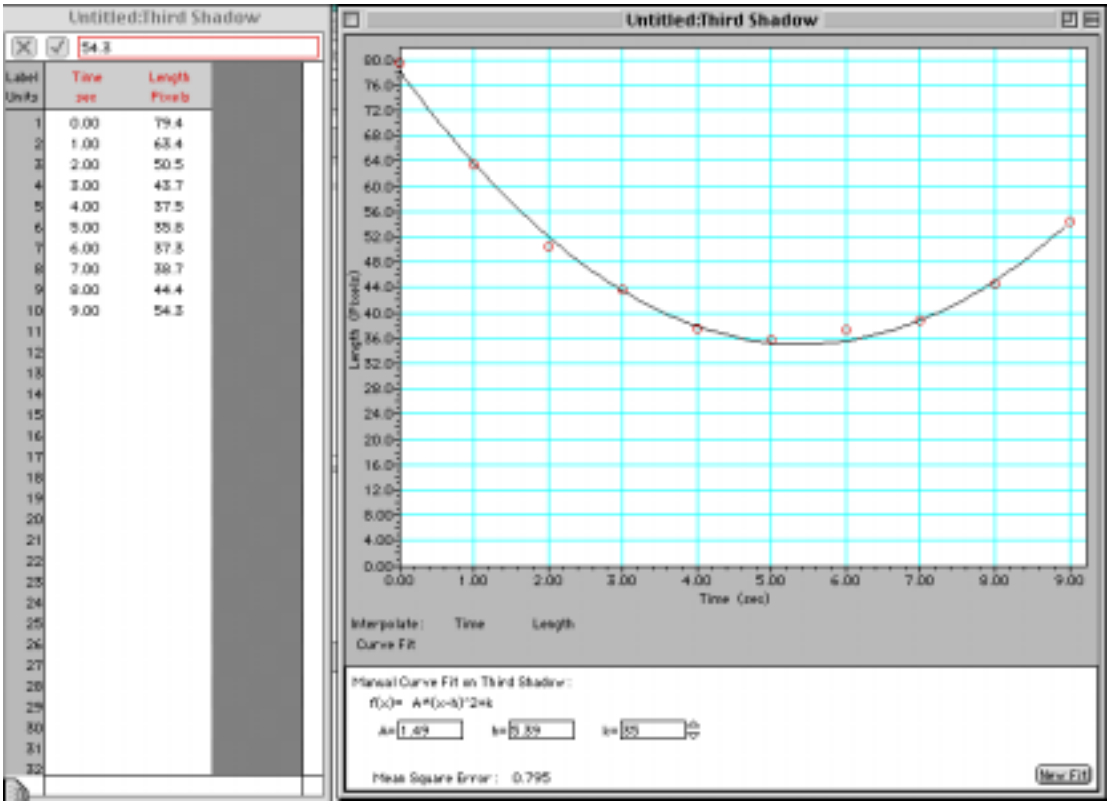

I. **Follow ups/extensions**:

There is a quiz specific to the lesson.

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Mathematics in the Shadows

Introduction

Long before recorded history, man has noticed the passage of the sun through the sky. There are many ancient structures that support this conclusion. One such structure is Stonehenge. Stonehenge is a monument constructed on the Salisbury Plain around 4000 years ago. The stones of Stonehenge are oriented to mark off certain dates of the year. The sun was also used to tell the time of day using a sundial. Egyptians made the earliest known sundials around 1500 B.C. The position of the sun indicates the time of day by casting a shadow on a dial that was part of the sundial. It is important to note that sundials are made specifically for the place in which they are used; that is, a sundial that works in San Antonio will not accurately tell the time in New York.

A simulation of the sun casting a shadow of an object was recorded. Watch the video clip and answer the following.

PART I. Analyzing the situation.

1. Identify the variables in this situation.

   Independent _____ Dependent _____

   Give reasons supporting your answer:_____

   _____

   _____

   _____

   _____

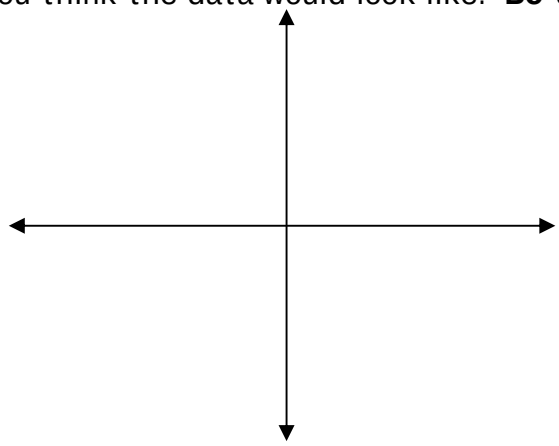
2. Describe what happened to the length of the shadow as time progressed.

   _____

   _____

   _____

3. Sketch a graph of what you think the data would look like. **Be sure to label your axes**.



4. Explain why a quadratic model would best fit this situation. **BE SPECIFIC!**

_____

_____

_____

5. Remember that the roots for an equation are the points where the graph intersects the X-axis. Should there be any roots for this first function? **Explain why or why not and when they would occur**. _____

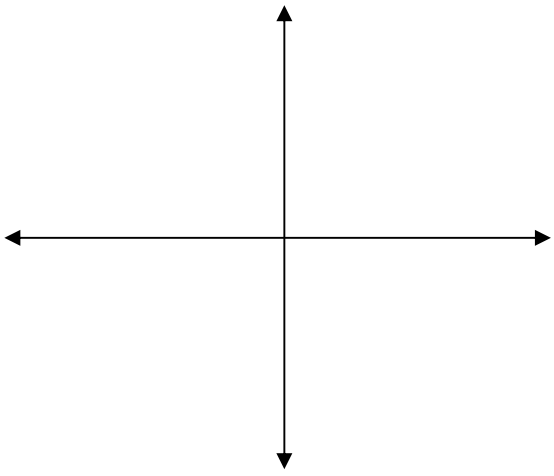
_____

_____

_____

Watch the second video clip and then answer the following.

6. Sketch a graph of what you think the data would look like. **Be sure to label the axes**.



7. Should there be any roots for this function? **Explain why or why not and when they would occur**._____

_____

_____

_____

8. Compare and contrast the graph of the second clip with the graph of the first clip. ***Be specific about the ways they are similar and the ways they are different.*** _____

_____

_____

_____

PART II. Collecting and analyzing the data

You will now collect the data for the **length of the shadow vs. time** using Scion Image. The movie is captured at a rate of 10 frames per second. **Be sure to fill-in the variables and the units in each table.** You will collect data at every **tenth frame**, starting at frame 1. You will do this for each of the three movies.

First Shadow

Second Shadow

Third Shadow

9. (a) Identify a **reasonable** domain and the range of the **data collected** for the first shadow.
DOMAIN: _____ **RANGE:** _____

(b) Identify a **reasonable** domain and the range of the **data collected** for the second shadow.
DOMAIN: _____ **RANGE:** _____

(c) Identify a **reasonable** domain and the range of the **data collected** for the third shadow.

DOMAIN: _____ **RANGE:** _____

Part III Analyzing the first shadow

- Launch Graphical Analysis
- Enter the data for the **first shadow**. Be sure to label the axes and units.
- Label this table as First Shadow by clicking on the title of the table.
- From Analyze, select Manual Curve Fit. Enter the equation
- $y = a*(x - h)^2 + k$.
- Change the **a, h, and k** values to get a good curve fit.
- When you have obtained a good curve fit, click OK - Keep Fit.

10. (a) Write the equation you obtained from this data: _____

(b) What is the mean sqr error for your equation? _____

Check with your teacher before you continue! _____

11. Write down the process by which you found the "***a***" value for your equation. **Be Specific!**

_____

_____

_____

12. What do the "**h**" and the "**k**" values represent?

_____

_____

_____

13. What is the real world meaning of the **vertex**?

_____

_____

_____

14. Use your equation in question 10(a) to find the length of shadow at **2 ½ seconds**. **Show your work**.

15. Use the **Interpolation** tool under the **Analyze** menu to find the point(s) in time when the shadow is **40 pixels** long. (Remember there may be two)

_____

_____

_____

Part IV Analyzing the second shadow

- Under data***, select new data set***.
- Label this set as Second Shadow by double clicking on the title of the table.
- Enter the data for the second shadow.
- Create a graph for this data by selecting ***window, new window, and graph.***
- Click on the y-axis and select the second data set y, and then click on the x-axis and select the second data set x.
- From Analyze, select Manual Curve Fit. Enter the equation $\mathbf{y = a*(x - h)^2 + k}$.
- Change the **a, h, and k** values to get a good curve fit.
- When you have obtained a good curve fit, click 

16. a. Write the equation you obtained from the second shadow data. _____
    b. What is the mean sqr error? _____

17. a. What is the vertex of the second equation? _____

    b. How is the vertex different from the first equation? _____

_____

_____

When talking about functions, there are certain points that are important to observe. One such point in a quadratic equation is the **minimum** (or the **maximum**). As its name indicates, the **minimum** is the point where the function has the **lowest y-value. Similarly,** the **maximum** refers to the point where the graph has the **highest y-value.**

18. Identify and use the minimum of your data and one other point on the graph to write an equation that models this data. (HINT: Remember how you wrote the equation for *Over Da Watta!*) Show Your Work!

19. Write and compare the **parameters** of the computer-generated equation (#16(a)) with your equation (#18).
    Computer-generated equation _____

    Your equation _____

20. Using the equation found in #18, determine the length of shadow at **2 ½ seconds**. **Show your work.**

21. Use the **Interpolation** tool under the **Analyze** menu to find the point(s) in time when the shadow is **40 pixels** long. _____

22.(a) Compare the graphs of the first and second shadows – how is the graph of the second shadow different from the graph of the first shadow?

_____

_____

_____

(b) Why did the difference(s) occur?

_____

_____

_____

Part V Analyzing the third shadow

Under data***, select new data set***.

- Label this set as Third Shadow by double clicking on the title of the table.
- Enter the data for the second shadow.
- Create a graph for this data by selecting ***window, new window, and graph.***
- Click on the y-axis and select the third data set y, and then click on the x-axis and select the third data set x.
- From Analyze, select Manual Curve Fit. Enter the equation $y = a*(x - h)^2 + k$.
- Change the **a, h, and k** values to get a good curve fit.
- When you have obtained a good curve fit, click OK - Keep Fit

23. (a) Write the equation you obtained from the third shadow data.

_____

(b) What is the mean sqr error? _____

24. (a) What is the vertex of the third equation?_____

(b) Why is the vertex different from the previous two vertices?

_____

_____

_____

25. Explain why the **"a"** values are similar?

_____

_____

_____

26. Use the **Interpolation** tool under the **Analyze** menu to find the point(s) in time when the shadow is **40 pixels** long using the data for the third shadow.

_____

_____

_____

27. Using the equation in question 24(a) to find the length of shadow at **2 ½ seconds**. **Show your work.**

28. Compare the second and third graphs, how is the third graph different from the second graph? ***Be specific.***

_____

_____

_____

29. What contributed to the change of the two graphs?

_____

_____

_____

30. What would happen to the graph if the light source were moved further down to the side becoming horizontal?

_____

_____

_____

31. What would happen if the light were directed horizontally at the object?

_____

_____

32. Are there any roots to any of the 3 equations? Explain your answer and give a possible explanation as to why it is so._____

_____

_____

_____