

In Perspective

Examining Rational Functions

**San Antonio
Technology in
Education Coalition**

Algebra II

*Lesson created by
Thomas Ledvorowski*





Introduction About the Mathematics

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A. **Critical Mathematics Explored in This Activity**

The principal topic of this activity is exploring the inverse proportionality that occurs in the concept of perspective and modeling that behavior through tables, graphs, and functions. This lesson is designed to expose the students to the function $f(x) = \frac{1}{x}$ and its characteristics.

B. **How Students Will Encounter the Concepts**

The students will use Scion Image (or NIH Imaging) and Graphical Analysis or a graphing calculator to examine and compare the percentage change in size of an object as that object is moved away from the viewer. They will use the tools of the software or graphing calculator to build a table of values and interpret these values graphically and algebraically.

C. **Setting Up**

Teacher Procedures (before class begins):

- Make sure both the **Scion Image** (or **NIH Imaging**) and **Graphical Analysis 2.0** programs are on every computer.
- Make sure the folder named “**Perspective**” containing the images is on every computer.
- Students should have access to calculators.

D. **Teacher Notes**

Objective:

The student will examine an inverse function by collecting and representing data in a table, on a graph, and as a function.

Teachable Moments:

- Students need to understand that Graphical Analysis uses the term Inverse Function to refer to a Rational Function.
- Inverse Function and the inverse of a function are different concepts and should not be confused.
- Your Computer has a calculator built in. This is located in the Accessories folder under the start menu.
- Introduce the concept of **Asymptotes**. See questions 5 and 6.
- Students may need some help calculating percents.

Connections:

This activity introduces the rational function. This activity will be revisited in the future to examine the rational function in depth. The concepts of Domain, Range, Independent and Dependent variables are extended. The asymptotic behavior of the rational functions will be introduced.

Classroom Management Tip

Students are to work in groups and share their time on the computer performing various tasks.

Prerequisite knowledge/skills:

- Use of graphing calculator or Graphical Analysis and Scion Image (or NIH Imaging).
- Understanding of Domain, Range, and Independent and Dependent Variables.
- How to calculate percents.
- Understanding of positive and negative correlation.

Performance based questions for assessment of student understanding:

- How did you determine the independent and dependent variables?
- What is the difference between the domain and range of the data collected and the real world situation?
- Stress questions 5 and 6 in the student activity.
- Will the object ever disappear?
- Can the distance be less than zero?

Teacher Supplementary Comments:

- The domain and range can be found for the data collected, the real world situation, and for the mathematical function. In the survey of functions we will be looking for the students to determine the differences between the domain and range **of the collected data and the real world situation only.**
- If Graphical Analysis is not available, use the graphing calculator to determine the regression equation using the statistical capabilities.
- If using a graphing calculator to find a regression equation, the TI-83 will recognize the rational function as a power function where the exponent will be negative.

Part I:

- It is presumed that the students already had some experience working with **Scion Image (NIH Imaging)**. If this is not the case, some of the basics may need to be taught to the students. *For example, when measuring the length of an object, holding the shift key while dragging the measuring tool will make the line vertical/horizontal.*
- The students will be viewing 10 pictures through **Scion Image (or NIH Imaging)**. These pictures can be opened one at a time or all at once. For some students it might be easier to open one picture at a time. The Window feature in the menu bar allows the student to bring to the front the image they wish to work on. It may help the student to click on the resize box to view the picture in its entirety.
- Pixels will be used to measure the height of the dominos. The first domino is 8cm from the camera, or viewer. Each successive picture has the domino on the right moving away from the camera in 8 cm increments.

Part II:

- It is presumed that the students already had some experience working with **Graphical Analysis**. The instructions are less explicit as to how to make changes and input data using **Graphical Analysis**. The teacher may need to remind students how to do these activities.
- The use of the **automatic curve fit** feature of **Graphical Analysis** is used at the end of the exercise to create the equation of the inverse function.

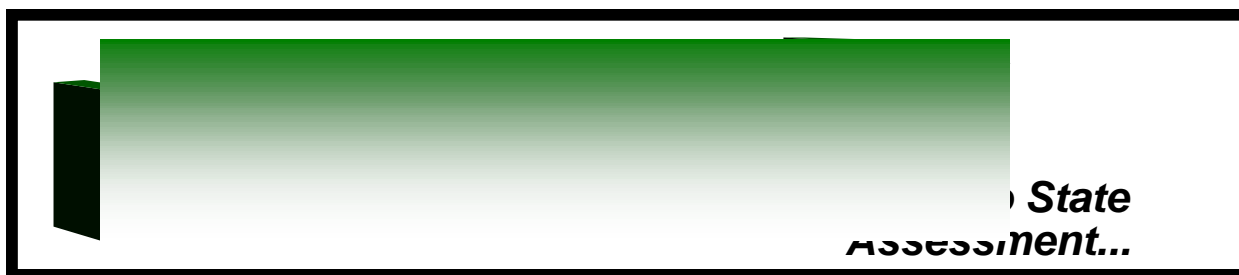
Follow ups/extensions:

A class discussion of the results of each group could follow this project. It will be important to bring out the general qualities of an inverse function. Some discussion

should take place as to the asymptotic behavior of the x- and y-axes, as well as the behavior of the function in the other quadrants of the coordinate plane.

Answers and Notes for Questions and Procedures:

1. B. As the distance from the viewer increases, the percentage decreases—proportionally not linearly.
2. The independent variable is the Distance from the viewer; the Dependent variable is the percentage.
3. The correlation of the data is negative.
4. The function is $f(x) = \frac{816}{x}$
5. As the object is moved closer than the original 8 cm to the viewer, it appears to get larger. Therefore as the distance decreases, the percentage increases.
6. As the object is moved to a distance that is very far, the object becomes very small, almost to disappear. Therefore, as the distance increases, the percentage decreases.
7. The domain is all numbers greater than (but not equal to) zero, because the object can be any distance from the viewer, except on top of it.
The Range will be any number greater than (but not equal to) zero, because the object will always have some percentage of the original size.



(b) Foundations of Functions

(1) The student uses properties and attributes of functions and applies functions to problem situations.

- (A) For a variety of situations, the student identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.
- (B) In solving problems, the student collects data and records results, organizes the data, makes scatterplots, fits the curves to the appropriate parent function, interprets the results, and proceeds to model, predict, and make decisions and critical judgments.

(c) Algebra and geometry: knowledge and skills and performance descriptions.

(1) The student connects algebraic and geometric representations of functions.

- (A) The student identifies and sketches graphs of parent functions, including linear ($y = x$), quadratic ($y = x^2$), square root ($y = \sqrt{x}$), inverse ($y = 1/x$), exponential ($y = a^x$), and logarithmic ($y = \log_a x$) functions.

(e) Rational functions: The student formulates equations and inequalities based on rational functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation.

- (1) The student uses quotients to describe the graphs of rational functions, describes limitations on the domains and ranges, and examines asymptotic behavior.
- (2) The student analyzes various representations of rational functions with respect to problem situations.
- (3) For given contexts, the student determines the reasonable domain and range values of rational functions, as well as interprets and determines the reasonableness of solutions to rational equations and inequalities.
- (4) The student solves rational equations and inequalities using graphs, tables, and algebraic methods.
- (5) The student analyzes a situation modeled by a rational function, formulates an equation or inequality composed of a linear or quadratic function, and solves the problem.
- (6) The student uses direct and inverse variation functions as models to make predictions in problem situations.

Student Activity ...

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Introduction

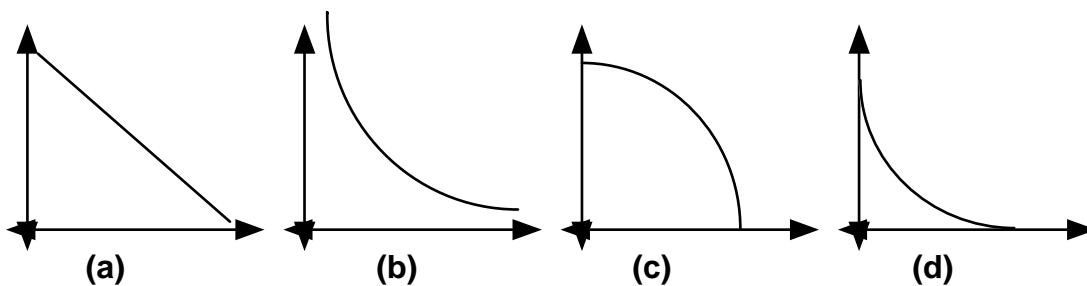
For centuries artists have been concerned with the phenomena called Perspective. If an artist wants to draw a sketch that shows depth of field, he or she has to follow the rules of perspective. Perspective is the illusion that as objects get further away from the viewer, they appear to get smaller. In this activity you will explore the mathematics of this concept.

For example, look at the picture of the torches to the right. Each torch is 6 feet tall. Yet, the second one appears to be shorter than the first, and the third even shorter yet. We want to determine how much shorter (using percentages) the second appears to be compared to the first and, again, how much shorter the third is from the first, and so on.

In this activity we will compare the percentage change of the apparent size of an object as it moves further away from the viewer.




1. **Make a Prediction:** Which graph below represents the relationship between the percent change of the original size as the distance from the viewer increases? **Explain your reasoning in complete sentences.**



Part I Collecting the Data

- Launch the program called **Scion Image**.
- Go to **File, Open** and open the images Domino 01 through Domino 10. (In the window, click on **Domino 01**, and then click the box next to **Open All** and then click **Open**.)
- Click on the image you want to measure.
- From the menu bar choose **Analyze, Options** and make sure that only the **Perimeter/length** box is checked.

Each picture contains the image of two dominos. The Domino on the left is exactly the same size as the domino on the right. The Domino on the right will be moved away from the “camera’s eye” 8 centimeters at a time.

- Access the Domino 01 Image
- Use the measuring tool  to find and record the height of the domino to the right. (click on the top of the domino, drag to the bottom, let go.)
- Click **Analyze, Measure**, then click **Analyze, Show Results**. ****Note that on the first image the domino on the right appears to be the exact same height as the one on the left. ****
- Repeat this for each of the 10 images. Use the table below to record your results.

- Calculate the percentage of the original size of the domino on the right for each image and record in the table.

Image Number	1	2	3	4	5	6	7	8	9	10
Distance from Camera										
Height in Pixels										
Percentage of Original size										

Part II: Analyzing Data

- Using the values of **distance and percentage**, determine which quantity is independent and which quantity is dependent.

1. Independent: _____
2. Dependent: _____

- Launch the program **Graphical Analysis**.
- Change the labels and include the units in the table.
- Enter in the data for distance and percentage from your table.

3. Describe the correlation of the data:

The data from this experiment takes represents a **Rational Function**, which is also called an Inverse Function in Graphical Analysis. That means that the percentage of the original size is inversely proportional to the distance the object is from the viewer. The parent function is described as $f(x) = \frac{1}{x}$ or $y = \frac{1}{x}$.

4. Using the **Automatic Curve Fit** in Graphical Analysis, choose the **Inverse Function** to fit a curve to your points. Write the equation that best represents the data you collected.

5. What happens to the percentage as the distance gets closer to zero?

6. What happens to the percentage as the distance increases to very large number?

7. From your observations, what would be a reasonable Domain and Range of this situation? Explain your reasoning.

Domain: _____

Range: _____