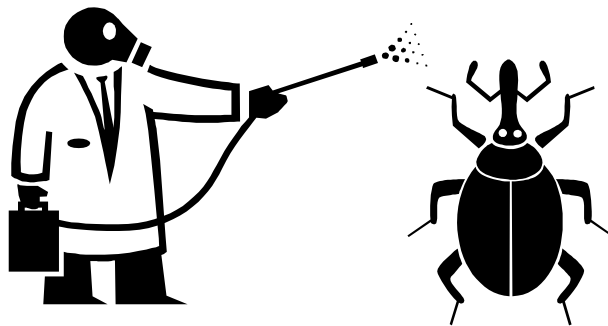


# San Antonio Technology In Education Coalition

## Exponential Functions

### Where Did The Buzz Bugs Go?



#### Patterns in Population Decay Exploring Patterns and Parameters

\*\*Adapted from the SI MMS module "Skeeters are Overrunning the World"

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# Where Did The Buzz Bugs Go?

## Teacher Notes

### A. Critical mathematics explored in this activity

In this activity, students look at how the size of a population increases or decreases over time. Students use **population models** to simulate different **growth rates** and study the effects of those rates on population size. These models and the mathematics behind them are used to predict future population changes.

For this activity, students should know **exponential notation**, how to use a spreadsheet or the STATS mode in the graphing calculator, how to graph on a **xy-coordinate system**, how to calculate **percent change**, and how to **evaluate** an equation.

### B. How students will encounter the concepts

Students explore what happens to a population over time by putting "buzz bugs" in a box and simulating buzz bug reproduction. Students will develop and use a mathematical model for population growth, determine the growth rate of a population, and graph and interpret an exponential function in the form  $y = a \cdot b^x$ .

### Setting Up

**Materials:** multicolored disks or candies (such as Skittles™) with a distinctive mark on one side, boxes with lids, colored pencils or markers, graphing calculators or Graphical Analysis program.

**Teacher Preparation:** As an introduction, explore human population growth. A good video to show is called "Human Population Growth" - it runs about 6 minutes. The student activity includes an introductory activity on human population growth.

### Teacher Notes

#### a) Objective:

- The objective of the lesson is to revisit the exponential model one more time to demonstrate that an exponential function can represent either growth or decay.

- The student will develop and use a mathematical model for population decay, determine the decay rate of a population, graph and interpret an exponential function to determine whether it is growth or decay model, and how to determine the difference between the two models.

**b) Teachable Moments**

- This Activity will allow you to review the pattern of the exponential functions, in particular the similarities between an equation showing growth and an exponential function showing decay.
- The students will need guidance in selecting the exponential curve fit since the previous time they worked with the exponential curve it was a growth.
- The teacher should reinforce the idea of asymptotes from the rational functions as to indicate that the population will never be below zero, hence the horizontal asymptote for the exponential function.
- The teacher has the opportunity now to talk about the difference in the equations, and how to determine when there will be growth, and when there will be decay by looking at the **b** value in the equation  $y = a \cdot b^x$

**c) Connections**

This lesson is studied here to explore the exponential function in depth, and study its properties such as the domain and range, its asymptotic behavior.

**d) Classroom Management Tips**

The students can use the same Skittles or M&M's they had previously used for the survey lesson.

**e) Pre-requisites**

- Students may need to be familiar with entering data in the calculator as well as how to find the regression equation for the data entered, calculate values either using the table, or the value feature.
- Students should be able to identify the difference between the exponential function and the quadratic models when selecting a model for this situation

**f) Questions**

Explain why a quadratic model would not be a good representation for this situation

**g) Supplementary Comments**

***Assessment***

One assessment idea is to give the students a data set and ask them to do the following:

- Write an equation to model it.

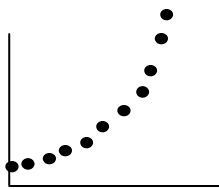
- Create a scatterplot for the data.
- Make predictions.
- Determine the reasonable domain and range.
- Determine what the y intercept for the data tells them.
- Determine what the x intercept for the data tells them.

**h) Follow-ups**

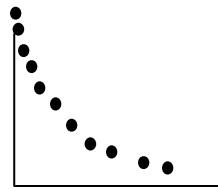
- Give each different colored buzz bug a different growth characteristic or initial population and see how changing those affects the growth equation.
- Look up current research about Human Population Growth.
- Find other things that are modeled with exponential functions.

**i) Answers**

1. The Equations will vary but should be similar.



D:  $x > 0$   
R:  $y \geq 2$



D:  $x > 0$   
R:  $0 \leq y \leq 100$

2. The pattern is nonlinear and population numbers are decreasing with time
3. The Attack...is a growing pattern, and the Where...is a decreasing pattern
4. They are similar but in the second case, b should be less than 1. The **b** value in the equation changed to a value between 0 and 1.
5. No, the domain and range should be the same.
6. Answers may vary. At that point the population might be at zero, or close to zero.
7. The population decreases after each shake. Each subsequent shake reduced the number of buzz bugs than the previous shake.
8. (a)  
(b)
9. The coordinate is (0, 2)
10. It represents the initial population.
11. NO, the nature of exponential functions is that they never cross the x-axis.
12. No, the growth rate is not constant.

## Student performance objectives - TEKS Correlation

### **(b) Foundations for functions: knowledge and skills and performance descriptions.**

- (1) The student uses properties and attributes of functions and applies functions to problem situations. Following are performance descriptions.
  - (A) For a variety of situations, the student identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.
  - (B) In solving problems, the student collects data and records results, organizes the data, makes scatterplots, fits the curves to the appropriate parent function, interprets the results, and proceeds to model, predict, and make decisions and critical judgments.

### **(c) Algebra and geometry: knowledge and skills and performance descriptions.**

- (1) The student connects algebraic and geometric representations of function
  - (A) The student identifies and sketches graphs of parent functions, including linear ( $y = x$ ), quadratic ( $y = x^2$ ), square root ( $y = \sqrt{x}$ ), inverse ( $y = 1/x$ ), exponential ( $y = ax^b$ ), and logarithmic ( $y = \log_a x$ ) functions.

### **(f) Exponential and logarithmic functions**

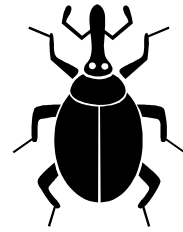
The student formulates equations and inequalities based on exponential and logarithmic functions, use a variety of methods to solve them, and analyze the solutions in terms of the situation.

- (2) The student uses the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describes limitations on the domains and ranges, and examines asymptotic behavior.
- (3) For given contexts, the student determines the reasonable domain and range values of exponential and logarithmic functions, as well as interprets and determines the reasonableness of solutions to exponential and logarithmic equations and inequalities.
- (5) The student analyzes a situation modeled by an exponential function, formulates an equation or inequality, and solves the problem.

## 6.5 Where did the Buzz Bugs Go?

### Patterns in Population Decay

#### Student Activity



#### Introduction

#### Exploration

To help make predictions in real-world situations, researchers often use experiments known as **simulations**.

In the following simulation, you will explore how the population of buzz bugs changes over time.

**Obtain a large, flat container with a lid and a sack of buzz bugs.**

#### Simulation Process:

- Place 90-100 buzz bugs in your container. This is the initial population.
- After closing the lid, shake the container.
- Open the lid, and **count** the number of buzz bugs with **marked side up**.
- Due to a chemical reaction, Buzz bugs are dying due to the light exposure. **Remove** each of the buzz Bugs that are marked side up.
- Record** the total number of buzz bugs now in the container in the table below. This is the end of one "shake." The end of each shake represents the end of one time period. The number of buzz bugs present at the end of a shake is the total population at that time. **(Remember that at shake 0, the number of buzz bugs was the number of Buzz Bugs in the container.)**
- Repeat** parts b-e until your population reaches zero or for 15 shakes.
- Do not record the shake when your population disappears.**

*Record your data in the table below:*

<i>Population</i>																	
<i>Shake Number</i>	0																

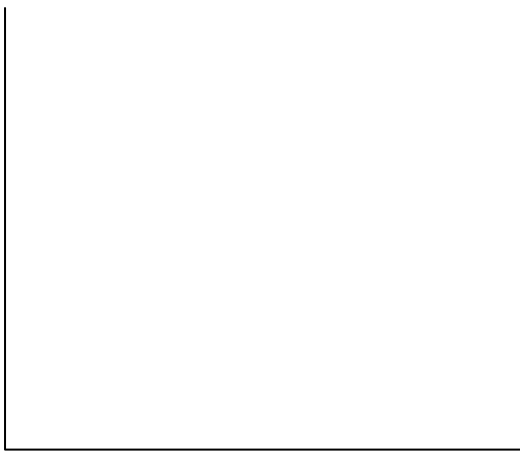
**Questions:**

- (a) Use your graphing calculator or Graphical Analysis program to create a scatterplot of the data you recorded.

(b) Sketch the graph you had previously obtained for the **Attack of the Buzz Bugs** in the space provided for **Graph A**, and sketch the graph you obtained from **Where did the Buzz Bugs Go?** in **Graph B**. Be sure to label the axis appropriately.

(c) Find the equation that best represents the data you collected using your graphing calculator.

**Graph A**



**Graph B**



Equation:

**Identify a reasonable Domain and range for this situation:**

Domain:

Range:

Equation:

Domain:

Range:

- Describe any pattern you see in your new data.
- 

- How does that compare to the pattern you had in the Attack of the Buzz Bugs?
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4. How does your *new equation* compare to the equation you had in the Attack of the Buzz Bugs? **Explain any change in the parameters.**

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5. When comparing the two graphs (*i.e. Attack of the Buzz Bugs and Where did the Buzz Bugs Go?*), are there any differences in the domain and range for the two functions? **Explain.**

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6. Use the equation you obtained for *Where did The Buzz Bugs Go?* to predict the number of buzz bugs after shake 20.

7. Consider your scatterplot as describing the change in population of buzz bugs over time. Use this idea to explain the shape of the graph.

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8. Assume that each light exposure represents a day, and the Buzz bugs population had been previously exposed to the sunlight **before you started recording** the population,

(a) When was the population 350? (That is, how many days before you began counting the buzz bugs in the experiment.)

(b) When was the population 485? (That is, how many days before you began counting the buzz bugs in the experiment.)

**Applying your knowledge of Exponential Functions and the data you collected.**

9. What is the coordinate of the y-intercept?\_\_\_\_\_
10. What information does the y-intercept tell you about this experiment?\_\_\_\_\_
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11. Is there an x-intercept for your graph?\_\_\_\_\_
- If yes, what is it?\_\_\_\_\_
- If not, why not?\_\_\_\_\_
- 

Note:

The **growth rate** of a population is the percent increase or decrease in the population between two time periods.

For example, the table below shows the population of rabbits in a cage over 3 years.

**Rabbit Population**

Year	Total Population
1992	16
1993	24
1994	36

The growth rate of the rabbit population from 1992 to 1993 is:

$$\frac{24 - 16}{16} = 50\% \text{ per year}$$

12. Is the growth (or decay) rate constant from shake to shake for the population of buzz bugs? Explain your response.