

The Attack of the Buzz Bugs:

Introduction to Exponential Functions



Patterns in Population Growth

***Adapted from the SIMMS module "Skeeters are Overrunning the World"*

A. Critical mathematics explored in this activity

In this activity, students look at how the size of a population increases or decreases over time. Students use **population models** to simulate different **growth rates** and study the effects of those rates on population size. These models and the mathematics behind them are used to predict future population changes.

B. How students will encounter the concepts

Students explore what happens to a population over time by putting "buzz bugs" in a box and simulating buzz bug reproduction. Students will develop and use a mathematical model for population growth, determine the growth rate of a population, and graph and interpret an exponential function in the form

$$y = a \cdot b^x$$

Setting Up

Materials: multicolored disks or candies (such as Skittles™) with a distinctive mark on one side, boxes with lids, colored pencils or markers, graphing calculators or Graphical Analysis program.

Teacher Preparation: As an introduction, explore human population growth. A good video to show is called "Human Population Growth" - it runs about 6 minutes. The student activity includes an introductory activity on human population growth.

Teacher Notes

a) **Objective:**

The student will develop and use a mathematical model for population growth, determine the growth rate of a population, and graph and interpret an exponential function.

b) **Teachable Moments**

- This Activity will allow you to review the patterns of the previous functions, in particular the differences between a quadratic and an exponential function.
- The students will need guidance in selecting a new curve fit, in this case the exponential function in order to obtain an equation from the calculator.
- The teacher should introduce the idea of asymptotes in order to explain that the population will never be below zero, hence there will be a horizontal asymptote for the exponential function.

c) **Connections**

The concept of domain and range will be extended. This lesson will be revisited as exponential decay when each function is studied in depth.

d) **Classroom Management Tips**

Because the candies will be used repeatedly, have another source of candy for the students to eat if you so desire.

Depending on the rate of growth and the number of Skittles available, it may be necessary to stop before 15 shakes.

e) **Pre-requisite knowledge/skills**

For this activity, students should know **exponential notation**, how to use a spreadsheet or the STATS mode in the graphing calculator, how to graph on an **xy-coordinate system**, how to calculate **percent change**, and how to **evaluate** an equation.

f) Questions

How do the domain and range compare to the linear and quadratic domain and range?

Why can't this situation be represented by either the linear or the quadratic function?

What might effect real population growth?

Emphasize questions 7,8,&9 in the student activity.

g) Supplementary Comments

Remind students that the axes in a real world situation are not x and y but are the corresponding independent and dependent variables. If letters are used, they need to be identified in a legend on the side.

Assessment

One assessment idea is to give the students a data set and ask them to do the following:

- Write an equation to model it
- Create a scatterplot for the data
- Make predictions
- Determine the reasonable domain and range
- Determine what the y intercept for the data tells them
- Determine what the x intercept for the data tells them

h) Follow-ups/extensions

- Give each different colored buzz bug a different growth characteristic or initial population and see how changing those affects the growth equation.
- Look up current research about Human Population Growth.
- Find other things that are modeled with exponential functions.

i) Answers

1. The pattern is nonlinear and population numbers are increasing with time.
2. Answers will vary.

The following table shows sample data for the 15 shakes.

Shake	Population	Shake	Population
0	2	8	40
1	3	9	61
2	4	10	87
3	6	11	114
4	7	12	164
5	12	13	225
6	17	14	308
7	26	15	424

1. Scatterplot should have axes correctly label and should be a nice exponential curve.
2. There is a nonlinear, curved pattern.
3. Sample response: The total population after shake 20 will be approximately 1300.
4. Answers may vary. Sample response: The population after shake 15 will be about 875, and after shake 16 about 1300, therefore it would take about 16 shakes.
5. The population increases after each shake. Each subsequent shake adds a greater number of buzz bugs than the previous shake.

6. Students should observe that the population size remains relatively unchanged for small shake numbers, but increases rapidly as the shake number increases.
7. Students should note that the graph is not linear since they get steeper as the number of shakes increases.
8. Sample response: bacteria
9. Answers will vary. One example: Buzz Bugs reproduce asexually whereas most animals and plants reproduce sexually.
10. Exponential
Answers may vary. For my sample data: $y=1.98*1.44^x$.
11. $X>0$. Shake numbers start at 0.
12. $Y\geq 2$. Our initial population is 2.
13. 2
14. The y-int. tells us our initial population.
15. NO, the nature of exponential functions is that they never cross the x-axis.
16. Answers may vary. For my sample data, 706,664 buzz bugs.
17. Answers may vary. For my sample data, 19 shakes.
18. No, the growth rate is not constant.

Student performance objectives - TEKS Correlation

- (b) Foundations for functions: knowledge and skills and performance descriptions.
- (1) The student uses properties and attributes of functions and applies functions to problem situations. Following are performance descriptions.
 - (A) For a variety of situations, the student identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.
 - (B) In solving problems, the student collects data and records results, organizes the data, makes scatterplots, fits the curves to the appropriate parent function, interprets the results, and proceeds to model, predict, and make decisions and critical judgments.
- (c) Algebra and geometry: knowledge and skills and performance descriptions.
- (1) The student connects algebraic and geometric representations of function. Following are performance descriptions.
 - (A) The student identifies and sketches graphs of parent functions, including linear ($y = x$), quadratic ($y = x^2$), square root ($y = \sqrt{x}$), inverse ($y = 1/x$), exponential ($y = ax$), and logarithmic ($y = \log ax$) functions.
- (f) Exponential and logarithmic functions: knowledge and skills and performance descriptions. The student formulates equations and inequalities based on exponential and logarithmic functions, uses a variety of methods to solve them, and analyzes the solutions in terms of the situation. Following are performance descriptions.
- (2) The student uses the parent functions to investigate, describe, and predict the effects of parameter changes on the graphs of exponential and logarithmic functions, describes limitations on the domains and ranges, and examines asymptotic behavior.

- (3) For given contexts, the student determines the reasonable domain and range values of exponential and logarithmic functions, as well as interprets and determines the reasonableness of solutions to exponential and logarithmic equations and inequalities.
- (5) The student analyzes a situation modeled by an exponential function, formulates an equation or inequality, and solves the problem.

The Attack of the Buzz Bugs:



Student Activity

Patterns in Population Growth

Introduction

Today more than 5.4 billion people inhabit the Earth. If each of us laid head to toe, we would make a chain long enough to wrap around the equator 250 times.

How many people can live on the Earth without destroying the environment? How many people can our planet successfully feed? Scientists are concerned with just these questions.

While you can't predict the future for certain, you can gather information about the past and present, find any existing patterns, and use these patterns to make predictions.

Graphs are one useful tool for determining existing patterns. For example, Figure 1 below shows the world population since 1650.

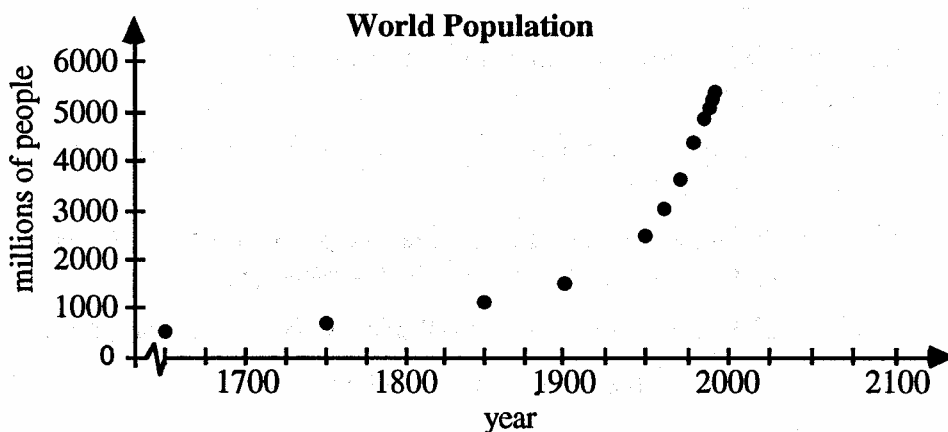


Figure 1: World population from 1650 A.D.

Questions

1. Describe the pattern you see in the world's population since 1650.

2. Based on the pattern you find, predict what you think the world population will be in the year 2050.

Exploration

Mathematical models allow researchers to make forecasts about population trends. For example, scientists at the United Nations predict a world population of at least 8.2 billion by the year 2020.

To help make predictions in real-world situations, researchers often use experiments known as **simulations**.

In the following simulation, you will explore how the population of buzz bugs changes over time.

Obtain a large, flat container with a lid and a sack of buzz bugs.

Simulation Process:

- Place 2 buzz bugs in your container. This is the **initial population**.
- After closing the lid, shake the container.
- Open the lid, and count the number of buzz bugs with marked side up.
- Buzz bugs reproduce asexually (by themselves). Reproduction is triggered when the marked side up of a buzz bug is exposed to light. Add one buzz bug to the container for each mark counted.
- Record the total number of buzz bugs now in the container in the table below. This is the end of one "shake." The end of each shake represents the end of one time period. The number of buzz bugs presented at the end of a shake is the total population at that time. (Remember that at shake 0, the number of buzz bugs was 2.)
- Repeat parts b-e for 15 shakes.

Describe in your own words the pattern you see for your data.

2. How did the number of buzz bugs in your population change during the simulation?

3. Consider your scatterplot as describing the change in population of buzz bugs over time. Use this idea to explain the shape of the graph.

4. Use your graphing calculator or Graphical Analysis to determine what type of Regression best fits the data collected. Keep in mind the previous functions we have studied.

a. What TYPE of function best fits the data? _____

b. Write the equation you found for your data.

5. What is a reasonable domain for this situation?

Why? _____

6. What is a reasonable range for this situation?

Why? _____

7. What is the y-intercept of your graph?

8. What information does the y-intercept give you?

9. Is there an x-intercept for your graph?_____

If yes, what is it?_____

If not, why not?

10. Is the growth rate constant from shake to shake for the populations of buzz bugs?

Explain your response.
