

Foundations of Functions

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*Lesson by Andy Sorensen and Thomas Ledvorowski
adapted from Letter of the Law*

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Introduction – About the Mathematics

A. Student Performance Objectives

(a) Basic Understanding

(3) Functions, equations, and their relationship. The study of functions, equations, and their relationship is central to all of mathematics.

Students perceive functions and equations as means for analyzing and understanding a broad variety of relationships and as a useful tool for expressing generalizations.

(5) Tools for algebraic thinking. Techniques for working with functions and equations are essential in understanding underlying relationships. Students use a variety of representations (concrete, numerical, algorithmic, graphical), tools, and technology, including, but not limited to, powerful and accessible hand-held calculators and computers with graphing capabilities and model mathematical situations to solve meaningful problems.

(6) Underlying mathematical processes. Many processes underlie all content areas in mathematics. As they do mathematics, students continually use problem-solving, computation in problem-solving contexts, language and communication, connections within and outside mathematics, and reasoning, as well as multiple representations, applications and modeling, and justification and proof.

(b) Foundations of Functions

(1) The student uses properties and attributes of functions and applies functions to problem situations. Following are performance descriptions.

(A) For a variety of situations, the student identifies the mathematical domains and ranges and determines reasonable domain and range values for given situations.

B. Critical Mathematics Explored In This Activity

The student should realize that each second of time (each data point in the domain) may be used only one time. This will lead into discussion of the formal mathematical definition of a function: Any set of ordered pairs is a function if each value of the domain is used only once. There are more rigorous definitions, but for our purposes, this simplified version suffices.

Functions are used everywhere, and we depend upon the fact that for a given independent variable, one and only one dependent value exists. The function we are working with in this exercise matches the independent values of time with the dependent values of distance from the motion detector at a given moment of time. Because it is not possible to use a moment of time twice, this makes a nice introduction to the function definition. The students are expected to demonstrate an understanding of what a function is and to determine one given a table, set, or graph, and their respective domain and range.

C. How the Student Will Encounter The Concepts

Students will be asked to create a graph by having the computer record their distance from a motion detector at a particular point in time. The graph they are to create should be in the shape of a letter. They will discover that most letters cannot be created by motion toward or away from the motion detector. As mentioned before, since time is ongoing and cannot be reused, letters with curves which turn back on themselves such as **B**, or that have a piece of the letter as a vertical line such as **H**, or that have part of the letter under another part of a letter like **A** cannot be accomplished with the required movements. As we discuss functions, we want students to have an understanding of the domain and range in functions and relations. Since the functions can have not two range values for a given domain value, we want the students to realize that in order to have two range points for a given domain value, the student must make the graph double back. This is not possible. Students will clearly see that only functions can be created using the motion detector. Thus as we move left to right on the x -axis, we will only have one value for y .

D. Connections

By now, students should be familiar with Domain and Range, Independent and Dependent variables, the use of the motion detector, and possibly depending on

their previous experience in Algebra I or Science, what letters describe a function.

This lesson will lead students from a visual picture of what a function is, to being able to determine a function or relation from a graph, table, or set of values. Connections to the vertical line test to verify a graphical representation and its connection to the tabular representation of two or more range values for a single domain value.

E. Assessments

Students can be orally quizzed for a verbal definition of a function, using letters of the alphabet not previously discussed as examples. Students should have to explain their reasoning.

Question 11 can be used for formal assessment for the activity as a homework assignment.

A Reflect and Apply assignment can be used as a follow up or as a formal assessment to make connections from their experience.

F. Setting Up (Note: This activity can be a successful teaching tool when done as an entire class, *or* when done in small groups)

Materials Needed

Logger Pro software

Computer with ULI or Lab Pro (or CBL with Graphing Calculator)

Motion Detector

Clamp or tape to secure the motion detector

File "2bornot2b"

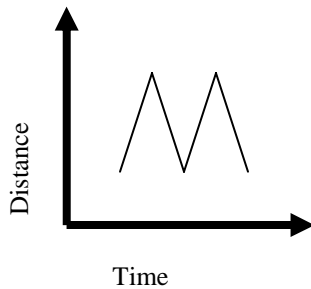
Students need to know how to launch Logger Pro and collect data. For those students who are comfortable with computers, the directions on the student page are sufficient. For others, you may need to model the activity or do this activity as a large group so that all students can participate in the discussion.

G. Answers for Student Activity page

- 1) Time and Distance
- 2) Time is the Independent variable and Distance is the Independent variable
- 3) $0 \leq \text{time} \leq 7$
- 4) $0 \leq \text{distance} \leq 2$

- 5) As you start the experiment, the time will begin to increase, as you move towards the motion sensor, the distance decreases. As you move away from the motion sensor the distance increases.
- 6) Move towards from the motion detector at a constant rate for a certain distance, then move away from the starting position to complete the same distance in the opposite direction.

7)



The units may vary from cases to case but be sure to emphasize that they indicate units, variables and to provide the domain and range for their respective graph.

- 8) Not it is not possible to create a graph representing the letter P. The reason is that you would need to be in two places at the same time which is impossible.
- 9) This could not be a function since it means that two or more values of the range are mapped to the same one value in the domain.
- 10)(a) No, because physically you would have to be in two places at the same time. It is not a function because two or more values of the range are mapped to the same value in the domain, and it fails the vertical line test.
(b) Yes it is.
- 11) Answers will vary. It is recommended you create your own rubric for this question.

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Survey of Functions

Student Activity

Instructions:

Launch the file *2bornot2b* on the computer (Make sure the ULI or Lab Pro is properly connected).

- 1.) What two quantities are we measuring?
- 2.) Identify the independent and dependent variables and justify your answer.
- 3.) What possible values for the domain would be graphed in this viewing screen?
- 4.) What possible values for the Range would be graphed in this viewing screen?
- 5.) What happens graphically (or to the graph) as you move towards the motion detector? How is each variable for this experiment affected by this movement?

Stand about five feet from the motion detector. Start collecting data. Move in the appropriate directions to make the letter "V" on the data collection screen (This may take several attempts).

6.) Describe the direction(s) of your motion as you created the letter "V".

7.) Describe, in step by step instructions, what direction(s) you would have to move in order to create the letter "M" ? Make a sketch in the empty space below of what will be graphed when you follow your instructions. Be sure to include units, and respective domain and range for your graph. Use the motion detector to verify your directions.

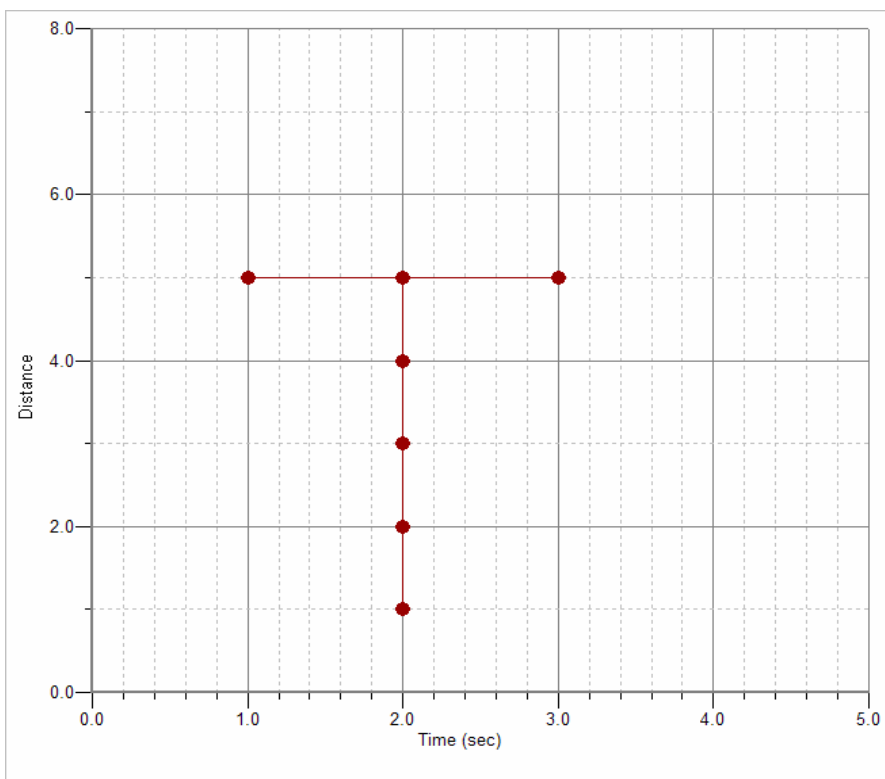


8.) Can you create a set of instructions for creating the letter "P"? If yes, list the steps necessary for creating the letter "P". IF NOT, Explain what it would imply physically with regard to your movements.

9.) Would the letter "P" be representative of a mathematical function? Justify your answer.

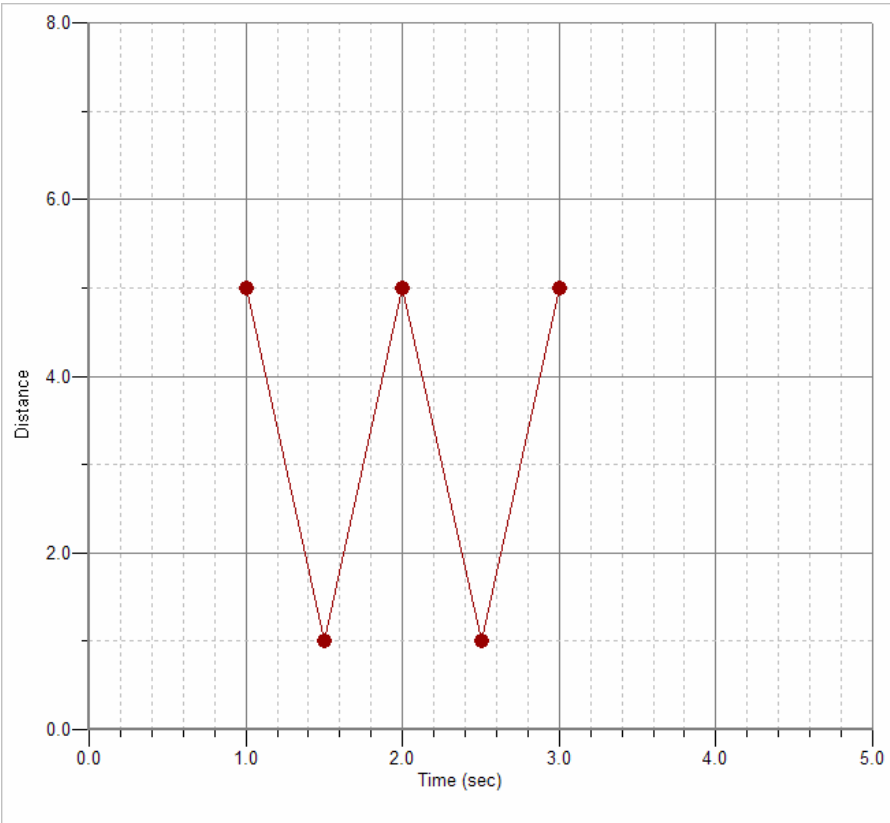
10.) Analyze the graphs below. Explain whether this graph could be replicated with the motion sensor? Why or why not? Is this a function? Why or why not? How can you determine mathematically if this is a function or not? Complete the table adjacent to the graph.

(a)



Time	Distance

(b)



Time	Distance

11) Write a paragraph which includes the following:

- The differences between the two graphs and provide a mathematical explanation of why each respective graph represents or does not represent a function.
- Make reference to the connection between the formal definition of a function by making use of the table, the graph, and applying your knowledge of drawing a function using the motion sensor.
- Provide a separate example (graph and table) of a letter that would represent a function, and a letter that would not represent a function explaining why each represents or not a function.