# Research on TEXTEAMS: Proportionality


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# Overview

- Importance of reasoning proportionally
- Effective professional development
- Types and levels of understanding

# A Problem

At school, Nickie and Max decided to have a contest to see who rode the fastest on their way home. Nickie rode 6 miles to her house in 24 minutes. Max rode 9 miles to his house in 32 minutes. Who rode faster? **How do you know?**

A proportion is a statement of equality between two ratios.

Symbolically, this is represented as $a:b = c:d$.

Proportional reasoning is the understanding of the underlying relationships found in a proportional situation, and the ability to work with such relationships. (From *Adding It Up: Helping Children Learn Mathematics*, National Research Council, 2001)

Embedded in proportionally reasoning is the concept of a multiplicative relationship.

Symbolically, this means all situations in which a proportional relationship exits can be modeled by the form $y = kx$.

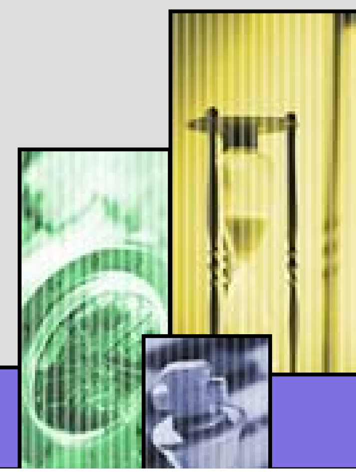
In other words, a proportional relationship exists when two amounts are related multiplicatively by a rate.

# Types of Proportional Problems

- Missing-value problems (e.g. cooking)
- Comparison problems (e.g. speed)
- Qualitative prediction and comparison problems (e.g. orange juice)

# Early research on students' understanding of proportionality

- Students in early to mid-adolescence are remarkably deficient in their ability to reason proportionally (Piaget, Karplus)
- Students are least likely to be successful when neither quantity of either ratio is a multiple of its corresponding quantity in the other ratio (Noelting)



# Two Types of Ratios

- Within-state ratios (e.g. time to time)
- Between-state ratios (e.g. time to distance)

Noelting, G. (1980). The development of proportional reasoning and the ratio concept. Part II: Problem-structure at successive stages; Problem-solving strategies and the mechanism of adaptive restructuring. Educational Studies in Mathematics, 11, 331-363.

# Areas of Proportional Reasoning

- Partitioning (involves concrete activity)
- Unitizing (cognitive)
- Quantities and change (interpret and operate)
- Rational numbers (meanings and operations)
- Ratio sense (intuition)
- Relative thinking (cognitive)

Page 3, Lamon, S. J. (1999b). More: In-depth discussion of the reasoning activities in "Teaching fractions and ratios for understanding". Mahwah, NJ: Lawrence Erlbaum Associates

# Proportional Reasoning Development

- A break from additive to multiplicative reasoning
- Accommodation of both covariance and invariance
- Multiple perspectives
- Complex units

# Proportional Reasoning Development (Lamon)

- Efficient in problem solving
- Effective in using decimals and fractions
- Deals with fraction density problems
- Understands covariation
- Identifies proportional and non-proportional relationships in real-life situations
- Adept at using scaling strategies
- Able to solve missing-value and comparison problems by reasoning, not rote procedure

# Effective Professional Development

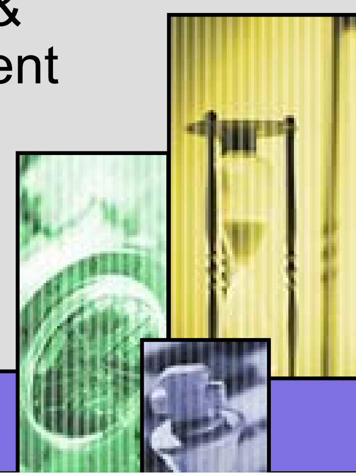
- Ensures collaboration
- Requires collective participation and implementation
- Focused on crucial problems of curriculum and instruction
- Conducted often enough and long enough to ensure progressive gains
- Contributes to professional habits and norms

Little, J. W. (1984). Seductive images and organizational realities in professional development. Teachers College Record, 86(1), 84-102.

# Effective Professional Development

- Active Learning
- Collective participation
- Coherence present (consistent with goals, standards and assessment)
- Reform oriented (conceptual), rather than traditional (learning teaching skills)
- Development over an extended period of time

Desimone, L. M., Porter, A. C., Garet, M. S., Yoon, K. S., & Birman, B. F. (2002). Effects of professional development on teachers' instruction: Results from a three-year longitudinal study. Educational Evaluation and Policy Analysis, 24(2), 81-112.



# Effective Professional Development

- Reflect and scrutinize teacher practices and beliefs
- Learn in the same method they are being asked to teach
- Continued opportunities to interact with the subject matter they teach

Borko, H. & Putnam, R. T. (1996). Learning to teach. In D.C. Berliner & R.C. Calfee (Eds.), Handbook of educational psychology (pp. 673-708). New York: Simon & Schuster Macmillan.

# Teachers' Understanding of Mathematics

- Conceptual - build upon experiences and prior understandings - rich in relationships
- Procedural - rules or algorithms - focuses on facts rather than connections
- Pedagogic Content Knowledge - how to teach specific content

# The Study

- What changes occurred in teachers' proportional reasoning abilities as a result of taking the TEXTEAMS Rethinking Middle School Mathematics: Proportionality Across the TEKS?

# The Participants

- 53 teachers from six different regions in Texas
  - Educational materials were offered as an incentive to participate
  - Teachers came from 25 campuses in 13 districts
  - 12 campuses had one teacher, 13 had two or more

# The Participants

**Type of certification**

| Multiple certificates | Elementary only | Elementary math specialist only | Secondary only | No certification | Special education |
|---|---|---|---|---|---|
| 20.75% | 35.85% | 9.43% | 28.30% | 1.89% | 3.77% |

**Grade level taught**

| 6th | 7th | 8th | Multiple | Other |
|---|---|---|---|---|
| 26.92% | 30.77% | 24.19% | 3.85% | 9.62% |

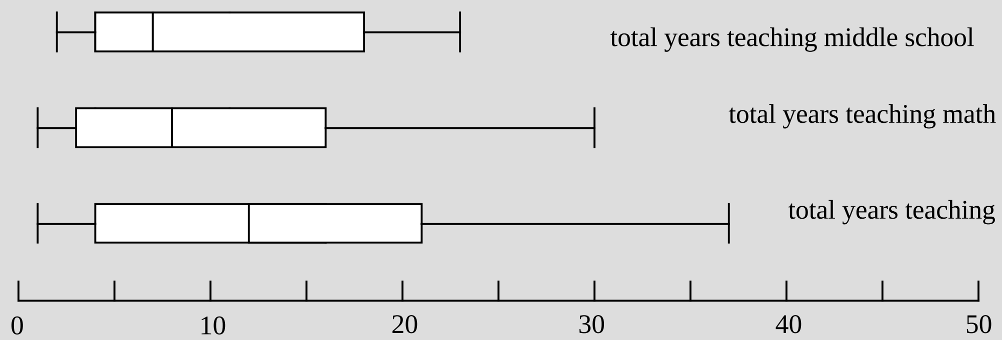
**Major**

| Education | Math | Multiple including education | Multiple including math | Non-math or education |
|---|---|---|---|---|
| 22.64% | 30.19% | 3.77% | 18.87% | 22.64% |

**Number of TEXTEAMS institutes attended**

| No previous | One previous | Multiple |
|---|---|---|
| 30.19% | 49.06% | 20.75% |

# The Participants




| | Mean | Median | Mode |
|---|---|---|---|
| Years Teaching Middle School | 8.8 | 7 | 1 |
| Years Teaching Math | 10.7 | 8 | 3 |
| Total Years Teaching | 12.9 | 12 | 2, 5 |

# The Workshops

- Seven workshops -
  - Six through ESC’s, one through the district
  - Five presented by in-house staff, two by the same outside consultant (a contributing author)

# The Instrument

- Four problems
  - 1 was designed to elicit recognition of proportional and non-proportional relationships
  - 2 comparison problem
  - 3 traditional missing-value problems
  - 4 was designed to differentiate between additive and multiplicative reasoning

# The Analysis

- Correctness of Answer
- Correctness of Reasoning
- Solution Strategy
- Representation
- Justification

# The Analysis

- Change from pre to posttest for the whole group
- Differences between mathematics and non-mathematics majors on the pretest? On the posttest?

# Findings for All Participants

| | Percent Correct Answers | | Percent Correct Reasoning | |
|---|---|---|---|---|
| Question | Pretest | Posttest | Pretest | Posttest |
| 1a | 96 | 100 | 92 | 96 |
| 1b | 92 | 100 | 64 | 92 |
| 2 | 77 | 87 | 75 | 87 |
| 3a | 89 | 94 | 91 | 96 |
| 3b | 70 | 83 | 72 | 83 |
| 4 | 81 | 94 | 74 | 92 |
| Total | 84 | 93 | 78 | 91 |

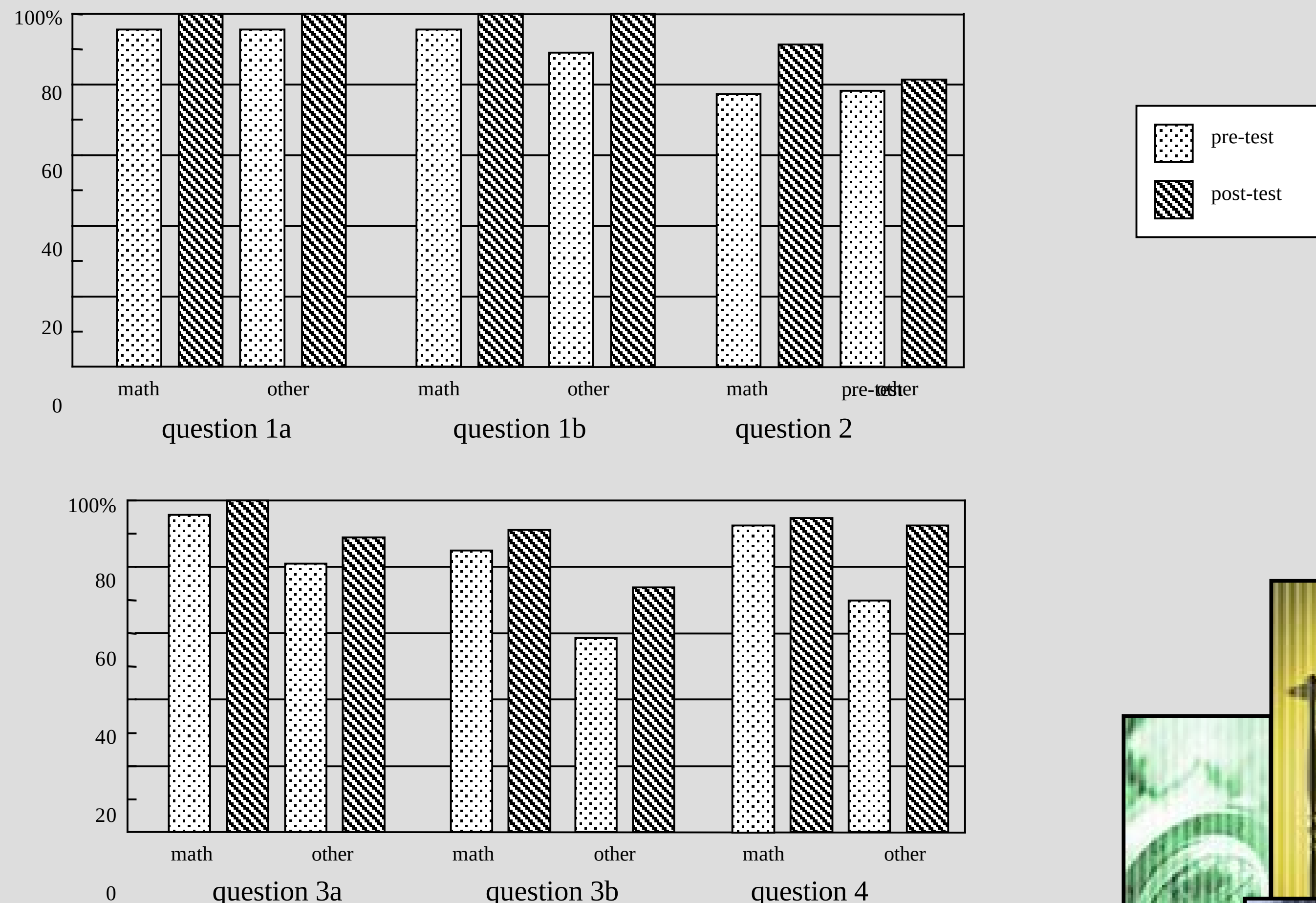
# Findings for All Participants

| | Percent of total number of instances of each strategy seen | |
|---|---|---|
| Strategy | Pretest | Posttest |
| Cross product | 48 | 38 |
| Unit rate | 14 | 17 |
| Equivalent ratios | 24 | 25 |
| Graphical | 1 | 8 |
| Scale factor | 13 | 30 |
| Justification | 12 | 16 |

# Participants Experience by Major

| | | Mean | Median | Mode |
|---|---|---|---|---|
| math major | Years Teaching Middle School | 8.7 | 6 | 6 |
| | Years Teaching Math | 11.3 | 7.5 | 3, 5 |
| | Total Years Teaching | 12.0 | 9.5 | 5 |
| other majors | Years Teaching Middle School | 8.8 | 7.5 | 1 |
| | Years Teaching Math | 10.1 | 10 | 2 |
| | Total Years Teaching | 13.4 | 12 | 2 |

# Findings for Math Majors vs. Non-math Majors: Percent Correct Answers




# Findings for Math Majors vs. Non-math Majors: Percent Correct Reasoning