

**DRAFT**

Mathematical Models with Applications

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# Fine Arts Module

## **Music**

# DRAFT

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## Music Unit Introduction

This unit explores the connections between mathematics and music. No musical background is required; however, access to someone who has some musical ability is helpful. One obvious connection music has with mathematics is rhythm; students learn about time values of notes and how they represent fractions of a measure. Students also explore sound waves with the graphing calculator. In addition, they investigate frequency, amplitude, and pitch. The unit concludes with the students playing music on bottles that they tune themselves.

An additional activity in this unit has the students investigate the geometry of music by creating a six-tone composition. Geometric transformations are given their music names and students use these transformations to compose a short musical piece which they then perform using the computer program provided with the materials.

The various activities in this unit together comprise a fairly complete introduction to the connections between mathematical concepts and music. However, some activities can be used as stand-alone activities while others can be omitted.

## Sequencing

The music unit can be taught as follows:

Geometric Music \*  
Rhythm  
Monochords  
Examining Sound  
Musical Bottles

The unit is designed to be taught using a class set of monochords. If you do not have (or want to make) a class set of monochords, you can adapt the activities that call for a class set of monochords into large group activities. The Playing a Tune Together activity in the Monochords section will probably need to be omitted.

\* Geometric Music is a stand-alone activity that fits into the curriculum in a variety of places. These placement options are discussed in more detail in the Geometric Music Teacher Notes.

The rhythm section of the music unit can also stand alone; however, it provides an excellent beginning to the rest of the unit.

## Unit Overview

The music unit addresses a different intelligence; it reaches the creative students more than the sequential learner. Expect that different students will take leading roles in group activities.

This unit is designed for the activities to be completed by groups of students. Create groups of 3-4 students that will work together for the entire unit. Because of varied backgrounds, if there are students that already read music try to put at least one musical student in each of the groups.

The first activity, Geometric Music, is one that could stand-alone or be used in other places within the fine arts module. The accompanying computer program will need to be installed well before beginning this activity; other software may need to be installed to run the program in addition to the program itself. Because of the nature of the program, there may be some technical issues that will need to be resolved with the help of a computer teacher or IT personnel. There are more details about these potential difficulties in the Geometric Music Teacher Notes portion of these materials. (If using the computer program is not possible, a keyboard can be used to perform the student compositions.)

If the Geometric Music activity is used as the first one, one or two of the compositions should be saved to be performed again using the monochords after Monochord Activity 2: The Well-Tempered Scale. Ask the composer to act as conductor or, if they want to be a performer, ask another classmate to act as conductor.

Assign the first note in each transformation to the first monochord, the second note to the second monochord, and so forth for all six notes. Have each group of students calculate the string lengths needed for the notes they will perform. The lowest note in the composition will have to be played as the open string with the other numbers assigned relative to the lowest note. Use the string length chart from the monochord section and list the note numbers starting at length 60 cm.

If your school teaches music theory, this activity can make a great cooperative effort. Invite the music theory students to perform some of their compositions for the mathematics students and/or have the music theory students use the Geometric Music compositions for ear training.

There are notes at the end of the Geometric Music section on how experienced music students can alter the composition. These advanced compositions will be difficult to grade, so use this idea with discretion.

The Rhythm Section could be taught independently of the rest of the unit, but is the beginning to a much more comprehensive look at the connections between mathematics and music. This section is the most musically demanding for the teacher so if the teacher is not familiar with musical notation, s/he may want to ask for help from the music department before beginning the unit.

Once there is a commitment to teach the entire unit, be sure to complete the Monochord Section and the Examining Sound Section. The Musical Bottles Section relies on having finished the preceding sections; it is the application section of the complete music unit. It could be omitted, but is the “fun” activity that requires using the knowledge gained from working through the unit.

If the choice is made to do Musical Bottles, tell the students that they need to begin saving soda bottles and cartons. Each group will need enough bottles for each note of their song (no more

than 7 bottles are necessary for any one song). Root beer and cream soda bottles are good because they also come in cartons; another option is Perrier water bottles. Plastic bottles are not as good an option as glass bottles because squeezing will alter the water level thus altering the pitch of the sound. Remind the students frequently to bring their bottles so that the groups will have their bottles ready when the bottles activity begins.

The major assessment for this unit is a test on the last day of the unit (a sample is included in these materials). However, some of the test questions will be given throughout the unit. The total of all the points from the end-of-section questions as well as the questions on the test will be combined to calculate the exam grade. There are no questions on the sample test from the Geometric Music activity; you may choose to add one to the test or to make the Geometric Music compositions an appropriate percentage of the test grade.

## Suggested and Required Materials

- student activity sheets for each activity, one per student
- 11” graph paper or Geometric Music Grid, one per student
- 3’x3’ (approximately) square of transparency film, one per student
- overhead pens, one per group
- GeometricMusic.jar file installed on computers (you may need to download and install JAVA as well)
- video of rhythmic performing group such as *Stomp*, *Riverdance*, or *Lord of the Dance* (optional)
- video *Donald Duck in Math Magic Land* (optional)
- one monochord for each group of 3-5 students
- graphing calculator for each student
- graphing calculator loaded with included programs, one per group
- data collection device, one per group
- cord to link graphing calculator and data collection device
- microphone and any required connections for the data collection device
- keyboard (can possibly be borrowed from music department; optional)
- bottles in six-bottle cartons, at most seven bottles per group
- disinfectant wipes for cleaning the tops of the drink bottles (ask the school nurse for these)
- large container to hold water for the bottle unit (a pickle bucket is perfect)
- a cup to use for pouring the water (a two-cup measuring cup that will hang off the bucket works well)
- lots of paper towels for cleaning up the water spills

## Prerequisites

The only prerequisite beyond basic mathematics, is that students have a familiarity with graphing calculators. All other skills are taught within the unit.

## TEKS Objectives

- (1) The student uses a variety of strategies and approaches to solve both routine and non-routine problems. The student is expected to:
  - (A) compare and analyze various methods for solving a real-life problem;
  - (B) use multiple approaches (algebraic, graphical, and geometric methods) to solve problems from a variety of disciplines; and
  - (C) select a method to solve a problems, defend the method, and justify the reasonableness of the results.
  
- (8) The student uses algebraic and geometric models to describe situations and solve problems. The student is expected to:
  - (A) use geometric models available through technology to model growth and decay in areas such as population, biology, and ecology; and
  - (B) use trigonometric ratios and functions available through technology to calculate distances and model periodic motion.
  
- (9) The student uses algebraic and geometric models to represent patterns and structures. The student is expected to:
  - (A) use geometric transformations, symmetry, and perspective drawings to describe mathematical patterns and structure in art and architecture; and
  - (B) use geometric transformations, proportions, and periodic motion to describe mathematical patterns and structure in music.

## TAKS Objectives

### Exit Level

**Objective 1: The student will describe functional relationships in a variety of ways.**

- A(b)(1) **Foundations for functions.** The student understands that a function represents a dependence of one quantity on another and can be described in a variety of ways.
- (B) The student [gathers and records data, or] uses data sets, to determine functional (systematic) relationships between quantities.
  - (D) The student represents relationships among quantities using [concrete] models, tables, graphs, diagrams, verbal descriptions, equations, and inequalities.

**Objective 2: The student will demonstrate an understanding of the properties and attributes of functions.**

- A(b)(2) **Foundations for functions.** The student uses the properties and attributes of functions.

- (C) The student interprets situations in terms of given graphs [or creates situations that fit given graphs].

**Objective 5: The student will demonstrate an understanding of quadratic and other nonlinear functions.**

A(d)(3) **Quadratic and other nonlinear functions.** The student understands there are situations modeled by functions that are neither linear nor quadratic and models the situations.

- (A) The student uses [patterns to generate] the laws of exponents and applies them in problem-solving situations.

**Objective 6: The student will demonstrate an understanding of geometric relationships and spatial reasoning.**

G(c)(1) **Geometric patterns.** The student identifies, analyzes, and describes patterns that emerge from two- and three-dimensional geometric figures.

- (B) The student uses the properties of transformations and their compositions to make connections between mathematics and the real world in applications such as tessellations or fractals.

**Objective 10: The student will demonstrate an understanding of the mathematical processes and tools used in problem solving.**

(8.14) **Underlying processes and mathematical tools.** The student applies Grade 8 mathematics to solve problems connected to everyday experiences, investigations in other disciplines, and activities in and outside of school. The student is expected to

- (A) identify and apply mathematics to everyday experiences, to activities in and outside of school, with other disciplines, and with other mathematical topics;
- (B) use a problem-solving model that incorporates understanding the problem, making a plan, carrying out the plan, and evaluating the solution for reasonableness; and
- (C) select or develop an appropriate problems-solving strategy from a variety of different types, including drawing a picture, looking for a pattern, systematic guessing and checking, acting it out, making a table, working a simpler problem, or working backwards to solve a problem.

(8.15) **Underlying processes and mathematical tools.** The student communicates about Grade 8 mathematics through informal and mathematical language, representations, and models. The student is expected to

- (A) communicate mathematical ideas using language, efficient tools, appropriate units, and graphical numerical, physical, or algebraic mathematical models.
- (8.16) **Underlying processes and mathematical tools.** The student uses logical reasoning to make conjectures and verify conclusion. The student is expected to
- (A) make conjectures from patterns or sets of examples and nonexamples; and
  - (B) validate his/her conclusions using mathematical properties and relationships.

# Geometric Music Grid

