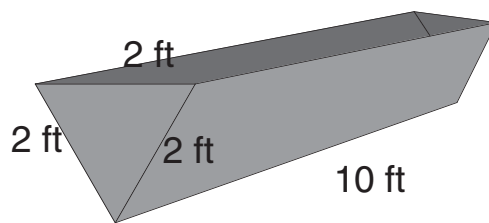


Walter and Juanita's Water Troughs

You have been hired as chief mathematician by a company named Walter and Juanita's Water Troughs. This company builds water troughs for various agricultural uses. The company has one design (see figure 1.). Your job is to perform mathematical analysis for the owners.

figure 1



1. A customer would like to know what the depth of the water is (in inches) if the trough has only 32 gallons in it.
2. The interior of the troughs must be coated with a sealant in order to hold water. One container of sealant covers 400 square feet. Will one container of sealant be enough to seal ten troughs? Why or why not?
3. Walter and Juanita would like to explore some minor modifications of their original design. They would like to know which change will produce a water trough that would hold more water—adding one foot to the length of the trough, making it 11 feet long, or adding three inches to each side of the triangular bases, making them 2 feet 3 inches on each side (see figures 2 and 3). Justify your answer.

figure 2

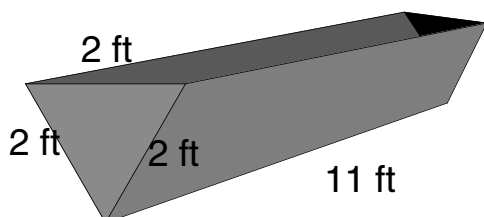
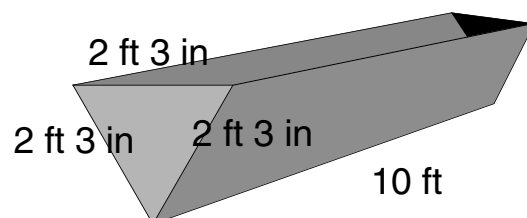


figure 3



Notes

Materials:

One calculator and straightedge per student

Geometry TEKS Focus:

(G.5) **Geometric patterns.** The student uses a variety of representations to describe geometric relationships and solve problems.

The student is expected to:

(D) identify and apply patterns from right triangles to solve meaningful problems, including special right triangles (45-45-90 and 30-60-90) and triangles whose sides are Pythagorean triples.

(G.8) Congruence and the geometry of size.

The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations.

The student is expected to:

(D) find surface areas and volumes of prisms, pyramids, spheres, cones, cylinders, and composites of these figures in problem situations.

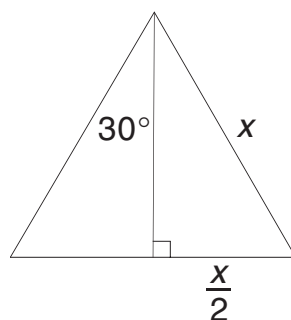
Teacher Notes

Scaffolding Questions:

- What important components of the trough are needed to solve the problems?
- What is your prediction for which trough will hold the most water in number 3? Justify your answer.

Sample Solutions:

1. To solve this problem, the area of the triangular base must be found in terms of an unknown side. If water is poured into the trough, the height of the water is a function of the side of the triangle. Consider the end of the trough that is an equilateral triangle.



Using 30-60-90 special right triangle properties or the Pythagorean Theorem, the altitude is $\frac{x\sqrt{3}}{2}$ ft.

Area of the base can be found by:

$$\text{Area} = \frac{1}{2}(x)\left(\frac{x\sqrt{3}}{2}\right)$$

$$\text{Area} = \frac{x^2\sqrt{3}}{4} \text{ ft}^2$$

The desired volume is given in gallons and must be converted to cubic feet because the measurements are given in feet.

$$32 \text{ gal.} \cdot \frac{1 \text{ ft}^3}{7.48 \text{ gal}} \approx 4.28 \text{ ft}^3$$

The volume of the tank is the area of the triangular base times the length of the tank.

$$\left(\frac{x^2\sqrt{3}}{4}\right)10 = 4.28 \text{ ft}^3$$

$$(x^2\sqrt{3})10 \approx 17.12$$

$$x \approx \pm\sqrt{0.988} \approx 0.994 \text{ ft}$$

The side length is 0.994 ft.

The depth of the water is the altitude of the base.

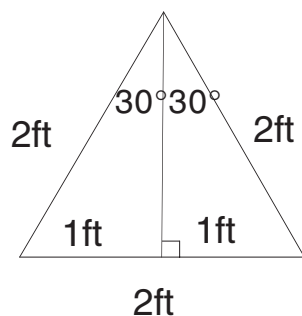
$$\text{altitude} = \frac{0.994\sqrt{3}}{2}$$

$$\text{altitude} \approx 0.861 \text{ ft}$$

$$\text{depth of water} \approx 10.3 \text{ in}$$

2. Find the interior surface area of one trough and multiply by 10.

Figure 2: First, find the area of the base (the ends of the trough). In order to do this, find the altitude of the triangle. The base is an equilateral triangle, so the altitude bisects the side and the intercepted angle. The angles of an equilateral triangle are all 60° .



Use the Pythagorean Theorem to find the altitude.

$$a^2 + b^2 = c^2$$

$$a^2 + 1^2 = 2^2$$

$$a^2 + 1 = 4$$

$$a^2 = 3$$

$$a = \pm\sqrt{3}$$

Additional Geometry TEKS:

(G.8) **Congruence and the geometry of size.** The student uses tools to determine measurements of geometric figures and extends measurement concepts to find perimeter, area, and volume in problem situations.

The student is expected to:

(A) find areas of regular polygons, circles, and composite figures;

(C) derive, extend, and use the Pythagorean Theorem; and

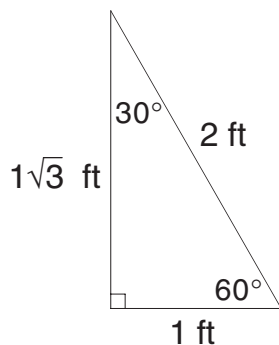
Connections to TAKS:

Objective 8: The student will demonstrate an understanding of the concepts and uses of measurement and similarity.

The altitude must be positive.

$$a = \sqrt{3}$$

Another approach to determining the altitude is to use 30-60-90 special right triangle properties. The hypotenuse is twice the shortest side. The side opposite the 60-degree angle is the shorter leg times $\sqrt{3}$.



The altitude is $1\sqrt{3} \approx 1.73$ ft.

The area of the triangular base may be found using the altitude and the base of the triangle.

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(2)(\sqrt{3}) = \sqrt{3} \approx 1.73 \text{ ft}^2$$

The surface area is the sum of the areas of the 2 ends and the 2 sides.

$$2\sqrt{3} + 2(20) \approx 43.64 \text{ ft}^2$$

The surface area of ten troughs is 10 times the surface area of one trough.

$$(10)43.46 = 434.6 \text{ ft}^2$$

Since the gallon of paint covers 400 ft^2 , there will not be enough sealant to seal ten troughs.

3. The volume of a right prism is the area of the base times the height.

The area of the base for the first case was found in problem 2.

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2}(2)(\sqrt{3}) = \sqrt{3} \approx 1.73 \text{ ft}^2$$

$$V = Bh = \sqrt{3}(11) \approx 1.73(11) \approx 19.05 \text{ ft}^3$$

For the second case, find the altitude of the right triangle base with side of 2.25 feet and the shorter leg, one-half of 2.25 or 1.125. Use the Pythagorean Theorem.

$$a^2 + b^2 = c^2$$

$$a^2 + (1.125)^2 = (2.25)^2$$

$$a^2 \approx 5.0625 - 1.266$$

$$a \approx \sqrt{3.8}$$

$$a \approx 1.95 \text{ ft}$$

Find the area of the base.

$$A = \frac{1}{2}bh$$

$$A \approx \frac{1}{2}(2.25)(1.95)$$

$$A \approx 2.19 \text{ ft}^2$$

Find the volume.

$$V = Bh$$

$$V \approx (2.19)(10)$$

$$V \approx 21.9 \text{ ft}^3$$

Adding 3 inches to each side of the base will produce a greater increase in volume than adding a foot to the distance between the bases.