

TASK 7.4.4: LORAN**Solutions**

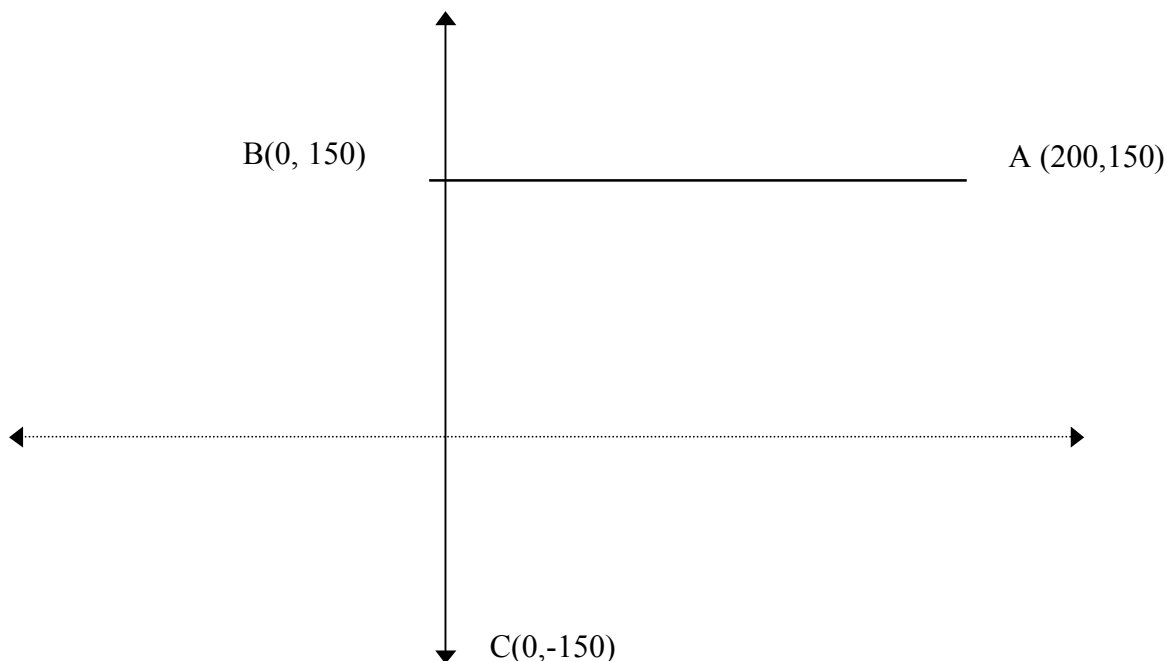
LORAN (Long Range Navigation) system is used to help navigators locate ships or planes. The Long Range Navigational system uses three broadcasting stations to determine the location of ships at sea. The stations send out a pulse. The navigator of the ship receives the pulses and determines the times taken to receive the signal. The distances from the three stations to the ship can be determined.

Three broadcasting stations are located at the points A , B , and C . A is located 200 miles east of B . C is located 300 miles south of B . The navigator sends a pulse and finds the following distances:

Point	A	B	C
Distance from Ship	209.6 miles	369.6 miles	569.6 miles

The stations are taken in pairs to serve as the foci of a hyperbola. The difference in the distances locates the ship on a hyperbola. The same calculations are taken using a different pair of stations. The second hyperbola is determined. The location of the ship is the intersection of the two hyperbolas. Find the equation of a hyperbola using A and B as the foci. Then write the equation of a second hyperbola using B and C as the foci. Use a graphing calculator to determine the location of the ship. **The ship is 331.66 miles east and 163.06 miles north of B .**

The coordinate axes may be placed anywhere. A possible location is to place the origin at the center of the hyperbola with foci B and C using the transverse and conjugate axes of the hyperbola. The coordinates of point A are $(200, 150)$. The center of the hyperbola determined by A and B is $(100, 150)$.



Consider the hyperbola with foci B and C.

c is half the distance between the foci or 150

The difference in the distances is $2a$.

$$2a = 569.6 - 369.6 = 200$$

$$a = 100$$

$$b^2 = c^2 - a^2 = 150^2 - 100^2$$

$$b = 111.8 \text{ miles}$$

The equations of the hyperbola is

$$\frac{y^2}{100^2} - \frac{x^2}{111.8^2} = 1$$

To graph on the calculator solve for y .

$$y = \pm 100 \sqrt{1 + \frac{x^2}{111.8^2}}$$

To determine the second hyperbola use B and A as foci.

$$c = 100$$

$$2a = 369.6 - 209.6 = 160$$

$$a = 80$$

$$b^2 = 100^2 - 80^2$$

$$b = 60 \text{ miles}$$

The center of the hyperbola is at (100,150)

The equations of the hyperbola is

$$\frac{(x - 100)^2}{80^2} - \frac{(y - 150)^2}{60^2} = 1$$

To graph on the calculator solve for y .

$$y = \pm 60 \sqrt{\frac{(x - 100)^2}{80^2} - 1} + 150$$

There are 3 intersections points.

The intersection points may be found by tracing or using the capability of a calculator to find the intersections points.

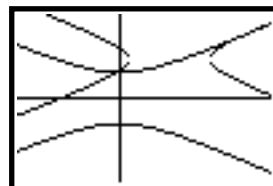
$$(-4.08, 100.6)$$

$$(331.66, 313.06)$$

$$(247.18, 242.66)$$

```
Plot1 Plot2 Plot3
\Y1=100\sqrt(1+X^2/11
1.8^2)
\Y2=-Y1
\Y3=60\sqrt((X-100)^2
/80^2-1)
\Y4=Y3+150
\Y5=-Y3+150
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```
WINDOW
Xmin=200
Xmax=300
Xscl=0
Ymin=-320
Ymax=320
Yscl=0
Xres=1
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Encourage a discussion on how can you decide which intersection to use. The ship must have been the specified distances from A, B, and C.

You may use the distance formula to determine which point satisfies the given distances.

Label the points.

M(-4.08,100.6) N(331.66,313.06) P(247.18, 242. 66)

Use the distance formula to find the distances to

A(200,150) B(0,150) C(0,-150)

MA= 210.01

MB=49.56

MC = 250.09

M is not the point because the distance to B should be about 396.6.

NA = 209.54

NB = 369.57

NC = 569.58

N is the required distance from each point.

PA = 103.98

P is not the point.

The point is the point (331.66, 313.06)

Is this the answer to the question?

Note that this point only has meaning in the selected coordinate system.

A description should be given in terms of one of the given points.

The ship is 331.66 miles east and 163.06 miles north of B.

Teaching notes

Present the problem to the group and allow participants time to solve in pairs.

As the students begin to work, circulate and ask questions to inspire their solutions.

Where might you position the origin?

It may be positioned anywhere. Choices might be at B, A, C, or midpoints of the segments AB or AC.

How many functions must be graphed to determine the location?

A hyperbola equation implies two functions. Thus, for two hyperbolas four functions must be graphed to determine the location.

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